

**Estudio Analítico - Gráfico  
de los  
Poliedros Arquimedianos**

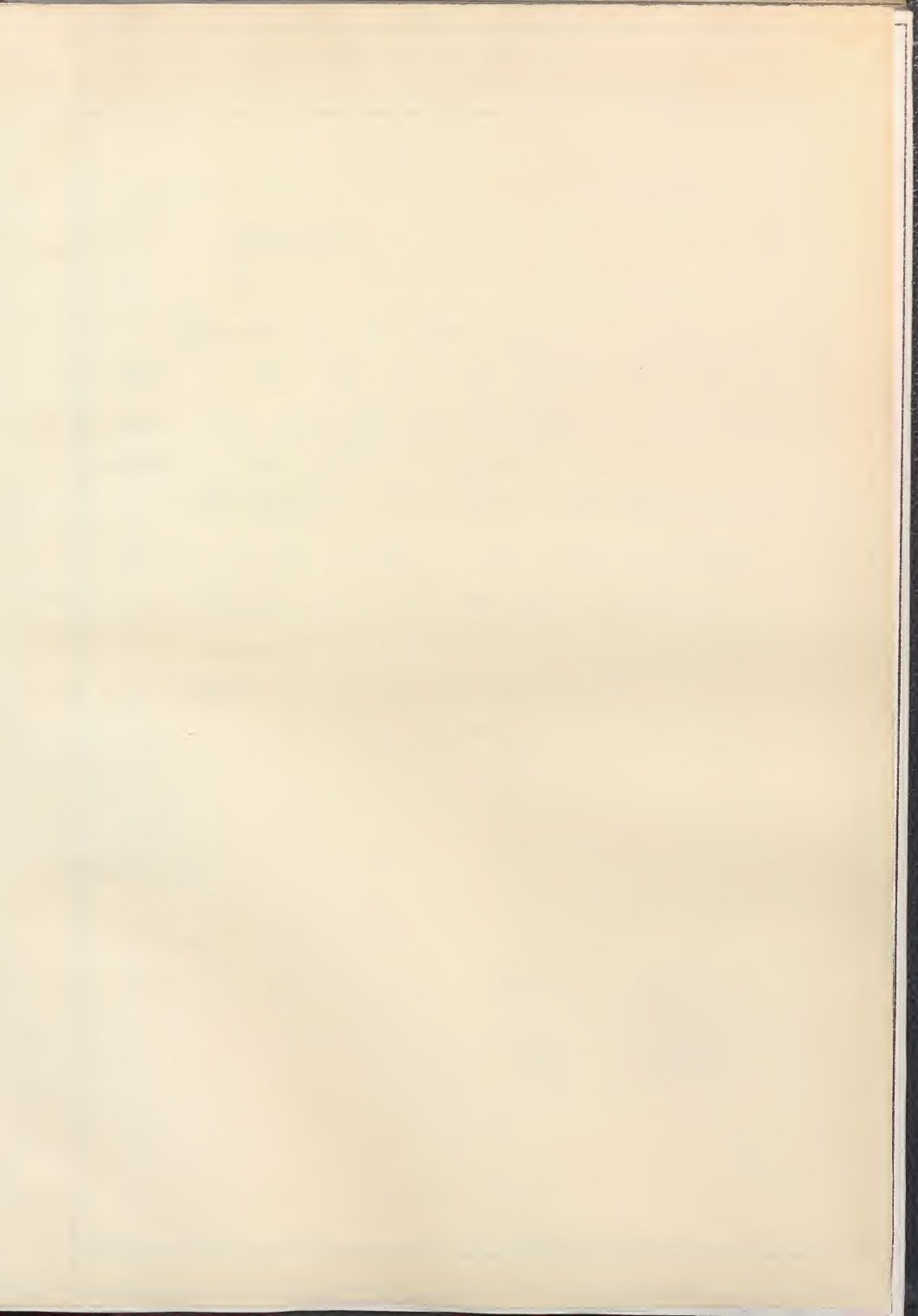
**LAMINAS 68 AL 43**

**VII**

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**Prof. T. Alvarez Peralto**









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ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimédiano VI en el que en cada vértice concurren un triángulo equilátero, dos cuadrados y un pentágono regular.

La longitud de su lado es de 24.6 mm, y las coordenadas de su centro O, son (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1.

DATOS

O (72, 72, 85) mm

$l_{VI} = 24.6 \text{ mm}$

A  
514  
ALV  
VII



514

31- 3- 73



CONSIDERACIONES PREVIAS

Loguiremos en el estudio de este arquimedianos, las directrices y formulas generales planteadas en el estudio del "Arquimedianos I", lamina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

$l$  = Arista del arquimedianos VI (dato del ejercicio)

$\alpha$  = Radio de la esfera circunscrita.

$b$  = Radio de la esfera tangente a las aristas.

$c_3$  = Radio de la esfera tangente a las caras triangulares.

$c_4$  = Radio de la esfera tangente a las caras cuadradas.

$c_5$  = Radio de la esfera tangente a las caras pentagonales.

$d_3$  = Radio de la circunferencia circunscrita a una cara triangular

$d_4$  = Radio de la circunferencia circunscrita a una cara cuadrada.

$d_5$  = Radio de la circunferencia circunscrita a una cara pentagonal.

$m$  = Radio de la circunferencia circunscrita al poligono obtenido al unir los extremos de las aristas de un ángulo sólido.

$\alpha_3$  = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimedianos

100

The first part of the book is devoted to a general introduction to the subject of the history of the world. It is a very interesting and useful book, and it is well worth reading. The author has done a very good job of summarizing the history of the world, and it is a very good book for anyone who is interested in the history of the world.

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no, que pasa por una arista de aquélla.

$\alpha_4$  = Ángulo rectilíneo del ~~del~~ diedro formado por una cara cuadrada, con el plano diametral del arquimedianos, que pasa por una arista de aquélla.

$\alpha_5$  = Ángulo rectilíneo del diedro formado por una cara pentagonal, con el plano diametral del arquimedianos, que pasa por una arista de aquélla.

$\varphi_{3-4}$  = Ángulo rectilíneo del diedro formado por una cara triangular y otra cuadrada

$\varphi_{4-5}$  = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra pentagonal.

$S$  = Superficie

$V$  = Volumen

#### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedianos, nos indica que se compone de 30 caras triangulares, 30 caras cuadradas y 12 pentagonales, todas regulares; 60 vértices y 120 aristas.

En cada vértice concurren, un triángulo, dos cuadrados y un pentágono, en el orden siguiente  $P_3 - P_4 - P_5 - P_4$ , es decir, las caras cuadradas no son consecutivas; por consiguiente concurrirán también 4 aristas del arquimedianos.

Así pues, tendremos que

The first of the two papers is by Dr. J. H. Hays, of the University of Chicago, and is entitled "The Pathology of the Heart in the Case of a Patient with a History of Rheumatism." The second paper is by Dr. J. H. Hays, of the University of Chicago, and is entitled "The Pathology of the Heart in the Case of a Patient with a History of Rheumatism." The first of the two papers is by Dr. J. H. Hays, of the University of Chicago, and is entitled "The Pathology of the Heart in the Case of a Patient with a History of Rheumatism." The second paper is by Dr. J. H. Hays, of the University of Chicago, and is entitled "The Pathology of the Heart in the Case of a Patient with a History of Rheumatism."

ORIGINAL ARTICLES

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and continued on p. 10



Let  $x$  and  $y$  be the two numbers. Then we have

$$x + y = 10$$

Also, we are given that

$$x^2 + y^2 = 58$$

From equation (1), we have

$$y = 10 - x$$

Substituting the value of  $y$  in equation (2), we get

$$x^2 + (10 - x)^2 = 58$$

Expanding the square, we get

$$x^2 + 100 - 20x + x^2 = 58$$

$$2x^2 - 20x + 100 = 58$$

$$2x^2 - 20x + 42 = 0$$

$$x^2 - 10x + 21 = 0$$

$$(x - 7)(x - 3) = 0$$

$$x - 7 = 0 \text{ or } x - 3 = 0$$

∴  $x = 7$  or  $x = 3$ . When  $x = 7$ ,  $y = 10 - 7 = 3$

When  $x = 3$ ,  $y = 10 - 3 = 7$

∴ The two numbers are 7 and 3.

Ans. The two numbers are 7 and 3.





y que la diagonal de un pentágono regular, de lado "l", es

$$DC = \frac{\sqrt{5} + 1}{2} l$$

Si en la figura 1, trazamos por E y F, puntos medios de los lados AD y AB, perpendiculares a estos lados, ambas se cortarán en O, centro de la circunferencia circunscrita al trapecio isósceles A-B-C-D; uniendo O con A, el segmento OA será el radio "m" de dicha circunferencia. Trazemos también por A, la perpendicular al lado DC, cuyo pie G nos determina el triángulo rectángulo A-D-G, recto en G.

De la figura se deduce:

$$AB = l \quad [1]$$

$$AD = BC = \sqrt{2} l \quad [2]$$

$$DC = \frac{\sqrt{5} + 1}{2} l \quad [3]$$

$$\begin{aligned} DG &= \frac{DC - AB}{2} = \left( \frac{\sqrt{5} + 1}{2} l - l \right) : 2 = \frac{1}{2} \left( \frac{\sqrt{5} + 1}{2} - 1 \right) l = \\ &= \frac{1}{2} \cdot \frac{\sqrt{5} - 1}{2} l = \frac{\sqrt{5} - 1}{4} l \end{aligned} \quad [4]$$

de [3] y [2] se deduce

$$\boxed{\operatorname{sen} \gamma} = \frac{DG}{DA} = \frac{\sqrt{5} - 1}{4} l : \sqrt{2} l = \frac{\sqrt{5} - 1}{4\sqrt{2}} = \boxed{\frac{\sqrt{10} - \sqrt{2}}{8}} \quad [5]$$

y siendo,  $\varphi = \gamma + \frac{\pi}{2}$  será  $\cos \varphi = -\operatorname{sen} \gamma$ , por lo que

$$\boxed{\cos \varphi} = -\operatorname{sen} \varphi = -\frac{\sqrt{10} - \sqrt{2}}{8} = \boxed{\frac{\sqrt{2} - \sqrt{10}}{8}} \quad \text{y por lo tanto} \quad [6]$$

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$$\boxed{\operatorname{sen} \varphi} = \sqrt{1 - \cos^2 \varphi} = \sqrt{1 - \left(\frac{\sqrt{2} - \sqrt{10}}{8}\right)^2} = \sqrt{1 - \frac{2 + 10 - 2\sqrt{20}}{64}} = \sqrt{\frac{64 - 12 + 4\sqrt{5}}{64}} =$$

$$= \sqrt{\frac{52 + 4\sqrt{5}}{64}} = \sqrt{\frac{13 + \sqrt{5}}{16}} = \boxed{\frac{\sqrt{13 + \sqrt{5}}}{4}} \quad [7]$$

De la figura se deduce:

$$AO = \boxed{m} = \frac{AF}{\cos \beta} = \boxed{\frac{l}{2 \cos \beta}} \quad \text{y también} \quad [8]$$

$$AO = \boxed{m} = \frac{AE}{\cos(\varphi - \beta)} = \boxed{\frac{\sqrt{2} l}{2 \cos(\varphi - \beta)}} \quad [9]$$

De [8] y [9]

$$\frac{l}{2 \cos \beta} = \frac{\sqrt{2} l}{2 \cos(\varphi - \beta)} \quad " \quad \cos(\varphi - \beta) = \sqrt{2} \cos \beta \quad "$$

$$\cos \varphi \cos \beta - \operatorname{sen} \varphi \operatorname{sen} \beta = \sqrt{2} \cos \beta \quad "$$

$$\cos \varphi \cos \beta - \operatorname{sen} \varphi \sqrt{1 - \cos^2 \beta} = \sqrt{2} \cos \beta \quad " \quad [10]$$

si hacemos en [10]  $\cos \beta = x$ ;  $\cos \varphi = p$ ;  $\operatorname{sen} \varphi = q$   
teniendo:

$$p x - q \sqrt{1 - x^2} = \sqrt{2} x \quad " \quad (p - \sqrt{2}) x = q \sqrt{1 - x^2} \quad "$$

$$p - \sqrt{2} = q \sqrt{\frac{1 - x^2}{x^2}} \quad " \quad \frac{p - \sqrt{2}}{q} = \sqrt{\frac{1}{x^2} - 1} \quad " \quad \left(\frac{p - \sqrt{2}}{q}\right)^2 = \frac{1}{x^2} - 1$$

$$\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1 = \frac{1}{x^2} \quad " \quad x^2 = \frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1} \quad " \quad \text{y de aquí:}$$

1.  $\frac{1}{x^2} = x^{-2}$

$$\frac{d}{dx} x^{-2} = -2x^{-3}$$

$$= -\frac{2}{x^3}$$

$$= -\frac{2}{x^3}$$

$$= -\frac{2}{x^3}$$

$$= -\frac{2}{x^3}$$

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$$= -\frac{2}{x^3}$$



$$\alpha = \cos \beta = \sqrt{\frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}} \quad [11]$$

valor que sustituido en [8] nos da

$$\begin{aligned} m &= \frac{l}{2 \cos \beta} = \frac{l}{2 \sqrt{\frac{1}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}}} = \frac{l}{\sqrt{\frac{4}{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}}} \times l = \\ &= \sqrt{\frac{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}{4}} \times l = \boxed{\frac{1}{2} \sqrt{\left(\frac{p - \sqrt{2}}{q}\right)^2 + 1}} \times l \quad [12] \end{aligned}$$

sustituyendo en [12] el valor de  $p = \cos \varphi = \frac{\sqrt{2} - \sqrt{10}}{8}$ , obtenido en [6], y el de  $q = \sec \varphi = \frac{\sqrt{13} + \sqrt{5}}{4}$  obtenido en [7], tendremos finalmente:

$$\begin{aligned} m &= \frac{1}{2} \sqrt{\left[\left(\frac{\sqrt{2} - \sqrt{10}}{8} - \sqrt{2}\right) : \frac{\sqrt{13} + \sqrt{5}}{4}\right]^2 + 1} \times l = \\ &= \frac{1}{2} \sqrt{\left[\frac{\sqrt{2} - \sqrt{10} - 8\sqrt{2}}{8} : \frac{\sqrt{13} + \sqrt{5}}{4}\right]^2 + 1} \times l = \frac{1}{2} \sqrt{\left(-\frac{7\sqrt{2} + \sqrt{10}}{8} : \frac{\sqrt{13} + \sqrt{5}}{4}\right)^2 + 1} \times l \\ &= \frac{1}{2} \sqrt{\left(-\frac{7\sqrt{2} + \sqrt{10}}{2\sqrt{13} + \sqrt{5}}\right)^2 + 1} \times l = \frac{1}{2} \sqrt{\frac{(7\sqrt{2} + \sqrt{10})^2}{4(13 + \sqrt{5})} + 1} \times l = \\ &= \frac{1}{2} \sqrt{\frac{98 + 10 + 14\sqrt{20}}{4(13 + \sqrt{5})} + 1} \times l = \frac{1}{2} \sqrt{\frac{108 + 28\sqrt{5}}{4(13 + \sqrt{5})} + 1} \times l = \end{aligned}$$

FL

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

Let  $y = \frac{1}{\sqrt{1-x^2}}$  then

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = \frac{-x}{(1-x^2)^{3/2}}$$

$$y = \frac{1}{\sqrt{1-x^2}} \Rightarrow \frac{dy}{dx} = \frac{-x}{(1-x^2)^{3/2}}$$

Integrating both sides we get  
 $\int \frac{dy}{dx} dx = \int \frac{-x}{(1-x^2)^{3/2}} dx$   
 $y = \frac{1}{\sqrt{1-x^2}} + C$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + C$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + C$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + C$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + C$$

$$\begin{aligned}
 &= \frac{1}{2} \sqrt{\frac{27 + 7\sqrt{5}}{13 + \sqrt{5}}} + 1 \times l = \frac{1}{2} \sqrt{\frac{27 + 7\sqrt{5} + 13 + \sqrt{5}}{13 + \sqrt{5}}} l = \frac{1}{2} \sqrt{\frac{40 + 8\sqrt{5}}{13 + \sqrt{5}}} \times l = \\
 &= \frac{1}{2} \sqrt{\frac{8(5 + \sqrt{5})}{13 + \sqrt{5}}} \times l = \sqrt{\frac{2(5 + \sqrt{5})}{13 + \sqrt{5}}} \times l = \sqrt{\frac{2(5 + \sqrt{5})(13 - \sqrt{5})}{13^2 - 5}} \times l = \\
 &= \sqrt{\frac{2(65 + 13\sqrt{5} - 5\sqrt{5} - 5)}{164}} \times l = \sqrt{\frac{60 + 8\sqrt{5}}{82}} \times l = \boxed{\sqrt{\frac{30 + 4\sqrt{5}}{41}}} \times l = \\
 &= 0,97460776 \dots l
 \end{aligned}$$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33), a este caso particular

$$\begin{aligned}
 \boxed{a} &= \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\sqrt{\frac{30 + 4\sqrt{5}}{41}} l\right)^2}} = \frac{1}{2\sqrt{1 - \frac{30 + 4\sqrt{5}}{41}}} \times l = \\
 &= \frac{1}{2\sqrt{\frac{41 - 30 - 4\sqrt{5}}{41}}} \times l = \frac{1}{2} \sqrt{\frac{41}{11 - 4\sqrt{5}}} \times l = \frac{1}{2} \sqrt{\frac{41(11 + 4\sqrt{5})}{11^2 - 16 \times 5}} l = \\
 &= \frac{1}{2} \sqrt{\frac{41(11 + 4\sqrt{5})}{41}} l = \boxed{\frac{1}{2} \sqrt{11 + 4\sqrt{5}}} \times l = 2,2329505 \dots l
 \end{aligned}$$

Para el caso del dibujo,  $a = 55,0 \text{ mm}$ , de donde  
 $l = 24,631 \text{ mm}$ .

1. The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations.

2. In the second part, we consider the case of a linear differential equation. It is shown that the problem is solvable in this case.

3. In the third part, we consider the case of a nonlinear differential equation. It is shown that the problem is solvable in this case.

4. In the fourth part, we consider the case of a system of differential equations. It is shown that the problem is solvable in this case.

5. In the fifth part, we consider the case of a partial differential equation. It is shown that the problem is solvable in this case.

6. In the sixth part, we consider the case of a system of partial differential equations. It is shown that the problem is solvable in this case.

7. In the seventh part, we consider the case of a differential equation with delay. It is shown that the problem is solvable in this case.

8. In the eighth part, we consider the case of a differential equation with stochastic perturbation. It is shown that the problem is solvable in this case.

9. In the ninth part, we consider the case of a differential equation with boundary conditions. It is shown that the problem is solvable in this case.

10. In the tenth part, we consider the case of a differential equation with initial conditions. It is shown that the problem is solvable in this case.



Radio "b" de la esfera tangente a las aristas.

Aplicando la fórmula general [3] (ver lám. 33), tendremos:

$$\boxed{b} = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{1}{2} \sqrt{11 + 4\sqrt{5}} \cdot l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{11 + 4\sqrt{5}}{4} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{10 + 4\sqrt{5}}{4}} \cdot l = \boxed{\frac{1}{2} \sqrt{10 + 4\sqrt{5}} \cdot l} = 2.17625090... \cdot l$$

(en dibujo:  $b = 53.6 \text{ mm}$ )

Radio "d<sub>3</sub>" de la circunferencia circunscrita a una cara triangular de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_3 = \frac{\sqrt{3}}{3} \cdot l} = 0.57735027... \cdot l$$

(en dibujo:  $d_3 = 14.2 \text{ mm}$ )

Radio "d<sub>4</sub>" de la circunferencia circunscrita a una cara cuadrada de lado "l"

Se demuestra en Geometría, es

$$\boxed{d_4 = \frac{\sqrt{2}}{2} \cdot l} = 0.70710678... \cdot l$$

(en dibujo:  $d_4 = 17.4 \text{ mm}$ ).

Radio "d<sub>5</sub>" de la circunferencia circunscrita a una cara pentagonal regular de lado "l"

The first part of the paper is devoted to the study of the  
 properties of the function  $f(x)$  defined by the equation  

$$f(x) = \frac{1}{2} (f(x-1) + f(x+1))$$
 for all  $x$  in the interval  $[0, 1]$ . It is shown that this function is  
 continuous and satisfies the boundary conditions  $f(0) = 0$  and  $f(1) = 1$ .

In the second part, we consider the problem of finding the  
 maximum value of the function  $f(x)$  on the interval  $[0, 1]$ . It is  
 shown that the maximum value is attained at  $x = \frac{1}{2}$  and is equal to  $\frac{1}{2}$ .

Finally, we discuss the question of the uniqueness of the  
 solution of the problem. It is shown that the function  $f(x)$  is the  
 only function satisfying the given conditions.

Se demuestra en Geometría, es

$$d_5 = \sqrt{\frac{5 + \sqrt{5}}{10}} l = 0,8506508... l$$

(en dibujo:  $d_5 = 27,0 \text{ mm}$ )

Radio "c<sub>3</sub>" de la esfera tangente a las caras triangulares regulares de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_3 &= \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\frac{1}{2} \sqrt{11 + 4\sqrt{5}} l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{11 + 4\sqrt{5}}{4} - \frac{1}{3}} \cdot l = \\ &= \sqrt{\frac{33 + 12\sqrt{5} - 4}{12}} \cdot l = \sqrt{\frac{29 + 12\sqrt{5}}{12}} l = \frac{\sqrt{29 + 12\sqrt{5}}}{2\sqrt{3}} l = \frac{\sqrt{\frac{40}{2}} + \sqrt{\frac{18}{2}}}{2\sqrt{3}} l = \\ &= \frac{\sqrt{20} + \sqrt{9}}{2\sqrt{3}} l = \frac{2\sqrt{5} + 3}{2\sqrt{3}} l = \frac{2\sqrt{15} + 3\sqrt{3}}{6} l = 2,15701985... l \\ &\quad \text{(en dibujo: } 58,1 \text{ mm.)} \end{aligned}$$

Radio "c<sub>4</sub>" de la esfera tangente a las caras cuadradas de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_4 &= \sqrt{a^2 - (d_4)^2} = \sqrt{\left(\frac{1}{2} \sqrt{11 + 4\sqrt{5}} l\right)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{\frac{11 + 4\sqrt{5}}{4} - \frac{2}{4}} \cdot l = \\ &= \sqrt{\frac{9 + 4\sqrt{5}}{4}} \cdot l = \frac{1}{2} \sqrt{9 + 4\sqrt{5}} l = \frac{1}{2} \left( \sqrt{\frac{10}{2}} + \sqrt{\frac{8}{2}} \right) l = \frac{1}{2} (\sqrt{5} + 2) l = \end{aligned}$$

Date	Page	Topic
<p>1. Let <math>f(x) = \frac{x^2 - 1}{x^2 + 1}</math> and <math>g(x) = \frac{x^2 + 1}{x^2 - 1}</math>.</p> <p>Find the domain of <math>f(x)</math> and <math>g(x)</math>.</p> <p>2. Find the range of <math>f(x)</math> and <math>g(x)</math>.</p> <p>3. Find the domain of <math>f(g(x))</math> and <math>g(f(x))</math>.</p> <p>4. Find the range of <math>f(g(x))</math> and <math>g(f(x))</math>.</p> <p>5. Find the domain of <math>f(f(x))</math> and <math>g(g(x))</math>.</p> <p>6. Find the range of <math>f(f(x))</math> and <math>g(g(x))</math>.</p> <p>7. Find the domain of <math>f(g(f(x)))</math> and <math>g(f(g(x)))</math>.</p> <p>8. Find the range of <math>f(g(f(x)))</math> and <math>g(f(g(x)))</math>.</p>		
Date	Page	Topic

$$= \left(1 + \frac{\sqrt{5}}{2}\right) l = 2,11803399... l$$

(en dibujo:  $c_4 = 52,2 \text{ mm}$ )

Radio " $c_5$ " de la esfera tangente a las caras pentagonales regulares de lado " $l$ "

Aplíandose la fórmula general [2], (ver lám. 33), tendremos:

$$c_5 = \sqrt{a^2 - (d_5)^2} = \sqrt{\left(\frac{1}{2}\sqrt{11+4\sqrt{5}} l\right)^2 - \left(\frac{\sqrt{5+\sqrt{5}}}{10} l\right)^2} = \sqrt{\frac{11+4\sqrt{5}}{4} - \frac{5+\sqrt{5}}{10}} \cdot l =$$

$$= \sqrt{\frac{55+20\sqrt{5}-10-2\sqrt{5}}{20}} \cdot l = \sqrt{\frac{45+18\sqrt{5}}{20}} l = \sqrt{\frac{9(5+2\sqrt{5})}{20}} l = \left[\frac{3}{2}\sqrt{\frac{5+2\sqrt{5}}{5}}\right] l =$$

$$= 2,0645729... l$$

(en dibujo:  $c_5 = 50,8 \text{ mm}$ )

Ángulo rectilíneo " $\alpha_3$ " del diedro formado por una cara triangular, con el plano diametral del arquimediiano que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\begin{aligned} \tan \alpha_3 &= \frac{2 c_3}{\sqrt{4 (d_3)^2 - l^2}} = \frac{2 \times \frac{2\sqrt{15} + 3\sqrt{3}}{6} l}{\sqrt{4 \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{2\sqrt{15} + 3\sqrt{3}}{3\sqrt{4 \times \frac{1}{3} - 1}} = \\ &= \frac{2\sqrt{15} + 3\sqrt{3}}{3 \times \sqrt{\frac{1}{3}}} = \frac{2\sqrt{15} + 3\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{45} + 9}{3} = \frac{6\sqrt{5} + 9}{3} = \boxed{2\sqrt{5} + 3} = \end{aligned}$$

*el*



Date	Page	No.
<p>1. * 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.</p>		
<p>2. * 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.</p>		
<p>3. * 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.</p>		

$$= 7, 47 \ 21 \ 35 \ 95 \dots$$

$$\lg \frac{t_7}{l} \alpha_3 = 0, 87 \ 34 \ 44 \ 8$$

$$\alpha_3 = 82^\circ \ 22' \ 32,5''$$

Ángulo rectilíneo " $\alpha_4$ " del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\boxed{\frac{t_7}{l} \alpha_4} = \frac{2 c_4}{\sqrt{4 (d_4)^2 - l^2}} = \frac{2 \times \left(1 + \frac{\sqrt{5}}{2}\right) l}{\sqrt{4 \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{2 + \sqrt{5}}{\sqrt{4 \times \frac{2}{4} - 1}} = \boxed{2 + \sqrt{5}} =$$

$$= 4, 23 \ 60 \ 67 \ 98 \dots$$

$$\lg 4, 23 \ 60 \ 67 \ 98 \dots = 0, 62 \ 69 \ 62 \ 9$$

$$\alpha_4 = 76^\circ \ 43' \ 2,9''$$

Ángulo rectilíneo " $\alpha_5$ " del diedro formado por una cara pentagonal regular, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\boxed{\frac{t_7}{l} \alpha_5} = \frac{2 c_5}{\sqrt{4 (d_5)^2 - l^2}} = \frac{2 \times \frac{3}{2} \sqrt{\frac{5 + 2\sqrt{5}}{5}} l}{\sqrt{4 \left(\sqrt{\frac{5 + \sqrt{5}}{10}} l\right)^2 - l^2}} = \frac{3 \sqrt{\frac{5 + 2\sqrt{5}}{5}}}{\sqrt{4 \times \frac{5 + \sqrt{5}}{10} - 1}} =$$

# [Title of the Document]

The following information is provided for your reference. This document contains details regarding the project and the work completed to date. The information is organized into sections for clarity.

The first section discusses the initial findings and the methodology used in the study. The second section provides a detailed analysis of the data collected.

The third section outlines the conclusions drawn from the research. The final section discusses the implications of the findings and suggests areas for future research.

The results of the study indicate that there is a significant correlation between the variables studied. The data suggests that the proposed model is effective in predicting the outcomes.

In conclusion, the study has provided valuable insights into the topic. The findings are consistent with previous research and offer new perspectives on the subject.

$$= \frac{3 \sqrt{\frac{5+2\sqrt{5}}{5}}}{\sqrt{\frac{10+2\sqrt{5}}{5}}} = \frac{3 \sqrt{\frac{5+2\sqrt{5}}{5}}}{\sqrt{\frac{5+2\sqrt{5}}{5}}} = \boxed{3}$$

$$l_3 \text{ } l_5 \text{ } \alpha_5 = 0,4771213\dots$$

$$\alpha_5 = 71^\circ 33' 54,2''$$

Ángulo rectilíneo  $\varphi_{3-4}$  del diedro formado por una cara triangular regular y una cuadrada

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{3-4}} = \alpha_3 + \alpha_4 = 82^\circ 22' 38,5'' + 76^\circ 43' 2,9'' =$$

$$= \boxed{159^\circ 5' 41,4''}$$

Ángulo rectilíneo  $\varphi_{4-5}$  del diedro formado por una cara cuadrada y una pentagonal regular.

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{4-5}} = \alpha_4 + \alpha_5 = 76^\circ 43' 2,9'' + 71^\circ 33' 54,2'' =$$

$$= \boxed{148^\circ 16' 57,1''}$$

Área lateral "S" del arquimedianos

Se compone de la suma de 20 caras triangulares re-

(2)  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$

$\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n}$  for  $|x| < 1$

By using the geometric series expansion, we can find the power series for  $\frac{1}{1-x^2}$ . We know that  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$ . Replacing  $x$  by  $x^2$ , we get  $\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n}$  for  $|x| < 1$ .

$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + \dots$

Similarly, we can find the power series for  $\frac{1}{1-x^3}$ . We know that  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$ . Replacing  $x$  by  $x^3$ , we get  $\frac{1}{1-x^3} = \sum_{n=0}^{\infty} x^{3n}$  for  $|x| < 1$ .

$\frac{1}{1-x^3} = 1 + x^3 + x^6 + x^9 + \dots$

Therefore, the power series for  $\frac{1}{1-x^2}$  is  $1 + x^2 + x^4 + x^6 + \dots$  and the power series for  $\frac{1}{1-x^3}$  is  $1 + x^3 + x^6 + x^9 + \dots$ .



gulares, 30 caras cuadradas y 12 caras pentagonales regulares, todas de lado "l"; la superficie será pues:

$$S = 20 \times \frac{\sqrt{3}}{4} l^2 + 30 l^2 + 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} l^2 = (5\sqrt{3} + 30 + 3\sqrt{25+10\sqrt{5}}) l^2$$

$$= (8,6602541 + 30 + 20,6457288) l^2 = 59,3059829... l^2$$

### Volumen "V" del arquimedianos

Se compone de la suma de 20 pirámides de base triangular regular y altura "C<sub>3</sub>"; de 30 pirámides de base cuadrada y altura "C<sub>4</sub>"; y finalmente de 12 pirámides de base pentagonal regular y altura "C<sub>5</sub>"; su volumen será pues:

$$\begin{aligned} V &= 20 \times \frac{\sqrt{3}}{4} l^2 \times \frac{C_3}{3} + 30 l^2 \times \frac{C_4}{3} + 12 \times \frac{\sqrt{25+10\sqrt{5}}}{4} l^2 \times \frac{C_5}{3} = \\ &= \frac{5\sqrt{3}}{3} l^2 \times \frac{2\sqrt{5}+3\sqrt{3}}{6} l + 10 l^2 \times \left(1 + \frac{\sqrt{5}}{2}\right) l + \sqrt{25+10\sqrt{5}} l^2 \times \frac{3}{2} \sqrt{\frac{5+2\sqrt{5}}{5}} l = \\ &= \frac{5}{18} (2\sqrt{45}+9) l^3 + (10+5\sqrt{5}) l^3 + \frac{3}{2} \sqrt{\frac{5+2\sqrt{5}}{5} \times (25+10\sqrt{5})} l^3 = \\ &= \frac{5}{6} (2\sqrt{5}+3) l^3 + (10+5\sqrt{5}) l^3 + \frac{3}{2} \sqrt{\frac{5+2\sqrt{5}}{5} \times (5+2\sqrt{5}) \times 5} l^3 = \\ &= \frac{5}{6} (2\sqrt{5}+3) l^3 + (10+5\sqrt{5}) l^3 + \frac{3}{2} \times (5+2\sqrt{5}) l^3 = \\ &= \frac{5(2\sqrt{5}+3) + 6(10+5\sqrt{5}) + 9(5+2\sqrt{5})}{6} l^3 = \frac{10\sqrt{5}+15+60+30\sqrt{5}+45+18\sqrt{5}}{6} l^3 = \end{aligned}$$

Date	Description
	<p>1. 10/10/2020 - 10/10/2020</p>
<p>10/10/2020</p>	<p>10/10/2020</p>
<p>10/10/2020</p>	<p>10/10/2020</p>
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<p>10/10/2020</p>	<p>10/10/2020</p>
<p>10/10/2020</p>	<p>10/10/2020</p>
<p>10/10/2020</p>	<p>10/10/2020</p>

$$= \frac{120 + 58\sqrt{5}}{6} \ell^3 = \left(20 + \frac{29\sqrt{5}}{3}\right) \ell^3 = 41,61532378... \ell^3$$

FIGURA CORPÓREA

Se obtiene por acoplamiento de 20 triángulos equiláteros, de lado 24,6 mm; de 30 cuadrados y 12 pentágonos regulares también de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren 1 triángulo, 2 cuadrados y 1 en pentágonos, estando alternados los cuadrados. ( $P_3 - P_4 - P_5 - P_4$ ).

En el cuadro sinóptico que damos a continuación, resumimos los resultados analíticos obtenidos anteriormente.



<div data-bbox="145 30 290 90" data-label="Text"> <p>100</p> </div>	<div data-bbox="393 30 777 90" data-label="Text"> <p>100</p> </div>
<div data-bbox="207 151 984 211" data-label="Text"> <p>100</p> </div>	
<div data-bbox="466 272 569 302" data-label="Text"> <p>100</p> </div>	
<div data-bbox="124 393 1004 665" data-label="Text"> <p>100</p> </div>	
<div data-bbox="569 725 673 756" data-label="Text"> <p>100</p> </div>	
<div data-bbox="124 846 1004 967" data-label="Text"> <p>100</p> </div>	
<div data-bbox="238 1134 372 1270" data-label="Image"> </div>	
<div data-bbox="145 1421 290 1481" data-label="Text"> <p>100</p> </div>	<div data-bbox="393 1421 777 1481" data-label="Text"> <p>100</p> </div>

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\frac{1}{2} \sqrt{11 + 4\sqrt{5}} \ell$	2, 23 29 57... $\ell$
$b$	$\frac{1}{2} \sqrt{10 + 4\sqrt{5}} \ell$	2, 17 62 51... $\ell$
$c_3$	$\frac{2\sqrt{15} + 3\sqrt{3}}{6} \ell$	2, 15 70 20... $\ell$
$c_4$	$1 + \frac{\sqrt{5}}{2} \ell$	2, 11 80 34... $\ell$
$c_5$	$\frac{3}{2} \sqrt{\frac{5 + 2\sqrt{5}}{5}} \ell$	2, 06 45 73... $\ell$
$d_3$	$\frac{\sqrt{3}}{3} \ell$	0, 57 73 50... $\ell$
$d_4$	$\frac{\sqrt{2}}{2} \ell$	0, 70 71 07... $\ell$
$d_5$	$\sqrt{\frac{5 + \sqrt{5}}{10}} \ell$	0, 85 06 57... $\ell$
$m$	$\sqrt{\frac{30 + 4\sqrt{5}}{41}} \ell$	0, 97 46 08... $\ell$
$\alpha_3$	$\frac{1}{2} \alpha_3 = 2\sqrt{5} + 3$	$\frac{1}{2} \alpha_3 = 7. 47' 21'' 36...$ $\alpha_3 = 82^\circ 22' 38,5''$
$\alpha_4$	$\frac{1}{2} \alpha_4 = 2 + \sqrt{5}$	$\frac{1}{2} \alpha_4 = 4. 23' 60'' 68...$ $\alpha_4 = 76^\circ 43' 29''$
$\alpha_5$	$\frac{1}{2} \alpha_5 = 3$	$\frac{1}{2} \alpha_5 = 3$ $\alpha_5 = 77^\circ 33' 54,2''$
$\varphi_{3-4}$	$\lg \varphi_{3-4} = -\frac{3 - \sqrt{5}}{2}$ *	$\lg \varphi_{3-4} = -0.38' 19'' 66...$ $\varphi_{3-4} = 159^\circ 5' 41,4''$
$\varphi_{4-5}$	$\lg \varphi_{4-5} = -\frac{\sqrt{5} - 1}{2}$ **	$\lg \varphi_{4-5} = -0.61' 80'' 34...$ $\varphi_{4-5} = 148^\circ 16' 57,1''$
$S$	$(5\sqrt{3} + 30 + 3\sqrt{25 + 10\sqrt{5}}) \ell^2$	59, 30 59 83... $\ell^2$
$V$	$(20 + \frac{29\sqrt{5}}{3}) \ell^3$	41, 61 53 24... $\ell^3$

\* Ver cálculos lám. 44, hoja 12 (reverso)

\*\* Ver cálculos lám. 44, hoja 14 (reverso)





PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 38, a la representación gráfica del arquimedeano VI.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias cuyo cálculo efectuaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " $l_v$ " del arquimedeano, cuya longitud es de 24,6 m m.

Calculemos previamente las siguientes magnitudes:

$$\begin{aligned}
 l_{VI} &= \text{Dato del ejercicio} = 24,6 \text{ m m} \\
 a &= 2,23 \ 29 \ 51 \dots \times 24,6 = 55,0 \text{ m m} \\
 b &= 2,17 \ 62 \ 51 \dots \times 24,6 = 53,6 \text{ m m} \\
 c_3 &= 2,15 \ 70 \ 20 \dots \times 24,6 = 53,1 \text{ m m} \\
 c_4 &= 2,11 \ 80 \ 34 \dots \times 24,6 = 52,2 \text{ m m} \\
 c_5 &= 2,06 \ 45 \ 73 \dots \times 24,6 = 50,8 \text{ m m} \\
 d_3 &= 0,57 \ 73 \ 50 \dots \times 24,6 = 14,2 \text{ m m} \\
 d_4 &= 0,70 \ 71 \ 07 \dots \times 24,6 = 17,4 \text{ m m} \\
 d_5 &= 0,85 \ 06 \ 51 \dots \times 24,6 = 21,0 \text{ m m}
 \end{aligned}$$

Antes de proceder al trazado gráfico, observemos en la lámina 38 que la proyección del arquimedeano, en el plano II, presenta una forma muy regular que permite obtener directamente dicha proyección.

Las propiedades geométricas de ella son:

Name	Address	City

- 1) Las caras 1 al 5 y 56 al 60, son pentágonos regulares de vértices alternados, centro en  $O$  y lado " $l$ ".
- 2) Los vértices 6 al 15 y 46 al 55 son vértices de un decágono regular de centro en  $O$  y lado " $l$ ".
- 3) Los vértices 26 al 35 son vértices de un decágono regular de cuyo lado y radio de su circunferencia circumscrita calcularemos posteriormente.

Teniendo presente lo expuesto, el orden de operaciones del trazado gráfico (lámina 38), es el siguiente:

1° Situar el centro  $O$ , de coordenadas 72, 72, 85 mm.

2° Dibujar en I, II y III las proyecciones de la esfera circumsrita, de radio 55 mm.

3° Representar en I, II y III las caras pentagonales opuestas 1 al 5 y 56 al 60, supuesto el poliedro colocado con dichas caras paralelas a II y uno de sus lados (3-4 en la superior, y 59-60 en la inferior) perpendicular a I (utilícese la cota " $c_5$ " en I y III, y la " $d_5$ " en II).

4° Representar en II los vértices 6 al 15 y 46 al 55, que son a su vez los de un decágono regular de centro en  $O$ , y lado " $l$ " (utilícese el radio  $r_1$ ), con la mitad de sus lados paralelos a los de los pentágonos 1 al 5 y 56 al 60.

5° Representar en II los vértices 26 al 35 que son a

<div data-bbox="103 15 331 105" data-label="Text"> <p>1</p> </div>	<div data-bbox="331 15 839 105" data-label="Text"> <p>2</p> </div>	<div data-bbox="839 15 1015 105" data-label="Text"> <p>3</p> </div>
<div data-bbox="103 105 1015 1421" data-label="Text"> <p>             The first part of the report is devoted to a description of the work done during the last year. It is divided into two main sections: the first section deals with the work done in the laboratory, and the second section deals with the work done in the field. The first section is divided into three parts: the first part deals with the work done in the laboratory, the second part deals with the work done in the field, and the third part deals with the work done in the laboratory. The second section is divided into two parts: the first part deals with the work done in the field, and the second part deals with the work done in the laboratory. The first part of the report is devoted to a description of the work done during the last year. It is divided into two main sections: the first section deals with the work done in the laboratory, and the second section deals with the work done in the field. The first section is divided into three parts: the first part deals with the work done in the laboratory, the second part deals with the work done in the field, and the third part deals with the work done in the laboratory. The second section is divided into two parts: the first part deals with the work done in the field, and the second part deals with the work done in the laboratory.           </p> </div>		
<div data-bbox="103 1421 331 1481" data-label="Text"> <p>4</p> </div>	<div data-bbox="331 1421 839 1481" data-label="Text"> <p>5</p> </div>	<div data-bbox="839 1421 1015 1481" data-label="Text"> <p>6</p> </div>



se ves los de un decágono regular de centro en  $O$  y radio " $r_3$ " (sensiblemente igual al " $a$ "), colocando sus lados perpendiculares a las bisectrices de los ángulos del decágono 6 al 15 ya trazado.

Con las operaciones 3.ª a 5.ª quedan representados en II todos los vértices del arquimedianos que se numerarán y unirán entre sí en el orden que se indica en la lámina. Para obtener en I las proyecciones de los vértices que faltan (ya hemos situados los 1 al 5 y 56 al 60), bastará determinar previamente las alturas a que se encuentran con respecto al centro  $O$ ; dichas alturas vienen dadas por las magnitudes " $f_1$ ", " $f_2$ " y " $f_3$ ", o por las " $g_1$ ", " $g_2$ " y " $g_3$ ", cuyos valores obtendremos posteriormente; trazando rectas paralelas a I, II, a las distancias anteriores, puede completarse fácilmente la proyección total en I, valiéndose de las proyecciones en II.

Conocidas las proyecciones en I y II, la de la III es inmediata y no necesita explicación.

Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que darán mayor exactitud a dicho trazado.

Altura " $n$ " de una cara triangular

The first of these is the fact that the  
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Se demuestra en Geometría es

$$n = \frac{\sqrt{3}}{2} l = 0,8660254... l$$

Apotema "k" de una cara pentagonal

Se demuestra en Geometría es (ver lám. 4, fórm. 39)

$$k = \sqrt{\frac{5 + 2\sqrt{5}}{20}} l = 0,6881910... l$$

Distancia "g," de los vértices 6 al 15 al plano de la cara pentagonal 1 al 5, y de los vértices 46 al 55 a la cara pentagonal 56 al 60.

Considerando en I la cara pentagonal 1 al 5, y la contigua cuadrada 3-4-12-13, que forman entre sí el ángulo  $\varphi_{4-5}$ , ya conocido, y cuyos respectivos planos son perpendiculares a 5, se deduce que la altura "g," buscada es la proyección sobre III del eje de la cara cuadrada, siendo el ángulo de proyección

$$\varphi_{4-5} - 90^\circ = 148^\circ 16' 57,1'' - 90^\circ = 58^\circ 16' 57,1'' \quad \text{de donde}$$

$$g_1 = \cos 58^\circ 16' 57,1'' \times l = 0,5257341... l$$

Desarrollo del cálculo anterior:

$$1. \quad \left( \frac{1}{2} + \frac{1}{3} \right) \left( \frac{1}{4} + \frac{1}{5} \right)$$

$$= \left( \frac{3}{6} + \frac{2}{6} \right) \left( \frac{5}{20} + \frac{4}{20} \right)$$

$$= \left( \frac{5}{6} \right) \left( \frac{9}{20} \right)$$

$$= \frac{5 \times 9}{6 \times 20}$$

$$= \frac{45}{120}$$

$$= \frac{3}{8}$$

Example 2: Find the value of  $\left( \frac{1}{2} + \frac{1}{3} \right) \left( \frac{1}{4} + \frac{1}{5} \right)$

Solution: We know that  $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

And  $\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} = \frac{9}{20}$

Therefore,  $\left( \frac{1}{2} + \frac{1}{3} \right) \left( \frac{1}{4} + \frac{1}{5} \right) = \frac{5}{6} \times \frac{9}{20} = \frac{45}{120} = \frac{3}{8}$

Example 3: Find the value of  $\left( \frac{1}{2} + \frac{1}{3} \right) \left( \frac{1}{4} + \frac{1}{5} \right)$

$$\lg. \cos 58^{\circ} 16' 57,1'' = \bar{7}, 72 \ 07 \ 63 \ 7 = \lg 0,52 \ 57 \ 31 \ 1 \dots$$

$$\cos 58^{\circ} 16' 57,1'' = 0,52 \ 57 \ 31 \ 1 \dots$$

Este valor se puede obtener exactamente, mediante el cálculo trigonométrico de los ángulos que intervienen, cuyos valores hemos deducido anteriormente, y cuyo cálculo desarrollamos a continuación.

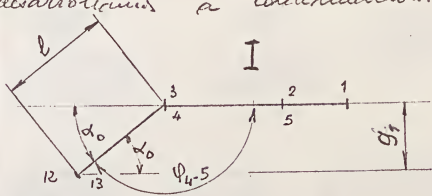


Figura 2

Sea (fig. 2) la proyección parcial en I del arquimedianos VI, que comprende la cara pentagonal 1 al 5 y la cuadrada 3-4-12-13

que forman entre sí el ángulo  $\psi_{4-5}$ , siendo  $\alpha_0$  el ángulo suplementario del  $\psi_{4-5}$ . La magnitud de la cota "g<sub>1</sub>" buscada, será pues

$$g_1 = l \operatorname{sen} \alpha_0 \quad [1]$$

pero siendo  $\psi_{4-5} = \alpha_4 + \alpha_5$  y  $\frac{1}{\tan} \alpha_4 = 2 + \sqrt{5}$ ;  $\frac{1}{\tan} \alpha_5 = 3$

tendremos

$$\begin{aligned} \frac{1}{\tan} \psi_{4-5} &= \frac{1}{\tan} (\alpha_4 + \alpha_5) = \frac{\frac{1}{\tan} \alpha_4 + \frac{1}{\tan} \alpha_5}{1 - \frac{1}{\tan} \alpha_4 \frac{1}{\tan} \alpha_5} = \frac{(2 + \sqrt{5}) + 3}{1 - (2 + \sqrt{5}) \times 3} = \\ &= \frac{5 + \sqrt{5}}{1 - 6 - 3\sqrt{5}} = \frac{5 + \sqrt{5}}{-5 - 3\sqrt{5}} = - \frac{5 + \sqrt{5}}{5 + 3\sqrt{5}} = - \frac{(5 + \sqrt{5})(3\sqrt{5} - 5)}{45 - 25} = - \frac{15\sqrt{5} + 15 - 25 - 5\sqrt{5}}{20} = \\ &= - \frac{10\sqrt{5} - 10}{20} = - \frac{\sqrt{5} - 1}{2} \end{aligned}$$

y de aquí se deduce, siendo



Date	Description	Amount
1917	Jan 1	100.00
1917	Feb 1	100.00
1917	Mar 1	100.00
1917	Apr 1	100.00
1917	May 1	100.00
1917	Jun 1	100.00
1917	Jul 1	100.00
1917	Aug 1	100.00
1917	Sep 1	100.00
1917	Oct 1	100.00
1917	Nov 1	100.00
1917	Dec 1	100.00
1917	Total	1200.00
1918	Jan 1	100.00
1918	Feb 1	100.00
1918	Total	200.00

$$\alpha_0 = \pi - \varphi_{4.5} \quad \text{y por lo tanto} \quad \tan \alpha_0 = - \tan \varphi_{4.5}, \quad \text{que}$$

$$\tan \alpha_0 = - \left( - \frac{\sqrt{5}-1}{2} \right) = \frac{\sqrt{5}-1}{2} \quad \text{y por consiguiente}$$

$$\boxed{\sec \alpha_0} = \frac{\tan \alpha_0}{\sqrt{1 + \tan^2 \alpha_0}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{1 + \left(\frac{\sqrt{5}-1}{2}\right)^2}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{1 + \frac{5+1-2\sqrt{5}}{4}}} =$$

$$= \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{4 + \frac{3-2\sqrt{5}}{2}}} = \frac{\frac{\sqrt{5}-1}{2}}{\sqrt{\frac{5-\sqrt{5}}{2}}} = \sqrt{\left(\frac{\sqrt{5}-1}{2}\right)^2 : \frac{5-\sqrt{5}}{2}} = \sqrt{\frac{5+1-2\sqrt{5}}{4} : \frac{10-2\sqrt{5}}{4}} =$$

$$= \sqrt{\frac{6-2\sqrt{5}}{10-2\sqrt{5}}} = \sqrt{\frac{3-\sqrt{5}}{5-\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})(5+\sqrt{5})}{25-5}} = \sqrt{\frac{15-5\sqrt{5}+3\sqrt{5}-5}{20}} =$$

$$= \sqrt{\frac{10-2\sqrt{5}}{20}} = \boxed{\sqrt{\frac{5-\sqrt{5}}{10}}} \quad \text{valor que sustituido en [1], nos da}$$

$$\boxed{g_1 = \sqrt{\frac{5-\sqrt{5}}{10}} \times l} = 0.52 \ 57 \ 31 \ 1... \ l$$

cuyo valor numérico aproximado es coincidente con el obtenido anteriormente.

Para el caso del dibujo, será:  $g_1 = 0.52 \ 57 \ 31 \ 1... \times 24.63 = 12.9 \ m$

Distancia "f," entre los dos planos paralelos a II, que contienen los vértices 6 al 15 y 46 al 55 respectivamente

Q. 1. A person starts from point A and goes to point B. The distance between A and B is 10 km. He goes from A to B at a speed of 5 km/h and returns from B to A at a speed of 4 km/h. Find the average speed for the whole journey.

Sol. Distance between A and B = 10 km  
 Speed from A to B = 5 km/h  
 Speed from B to A = 4 km/h

Time taken to go from A to B =  $\frac{\text{Distance}}{\text{Speed}} = \frac{10}{5} = 2$  hours  
 Time taken to return from B to A =  $\frac{10}{4} = 2.5$  hours

Total distance = 10 km + 10 km = 20 km  
 Total time = 2 hours + 2.5 hours = 4.5 hours

Average speed =  $\frac{\text{Total distance}}{\text{Total time}} = \frac{20}{4.5} = \frac{40}{9}$  km/h

∴ The average speed for the whole journey is  $\frac{40}{9}$  km/h.

Q. 2. A train starts from station A and goes to station B. The distance between A and B is 120 km. It starts at 10:00 AM and reaches B at 12:30 PM. Find the speed of the train.

Sol. Distance between A and B = 120 km  
 Time taken = 12:30 PM - 10:00 AM = 2.5 hours

Speed =  $\frac{\text{Distance}}{\text{Time}} = \frac{120}{2.5} = 48$  km/h

Se obtiene por diferencia de las alturas " $C_5$ " y " $g_1$ ", ya calculadas.

Se puede simplificar

$$\boxed{f_1} = 2(C_5 - g_1) = 2 \times \left( \frac{3}{2} \sqrt{\frac{5+2\sqrt{5}}{5}} - \sqrt{\frac{5-\sqrt{5}}{10}} \right) \cdot l = \left( 3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5-\sqrt{5}}{10}} \right) l =$$

$$= 2 \times (2,0645729... - 0,5257311...) l = 3,0776836... l$$

Para el caso del dibujo, será:  $f_1 = 3,0776836... \times 24,63 = 75,8$   
(véase simplificación al dorso)

Radio " $r_1$ " de la circunferencia circunscrita al decágono regular de lado " $l$ " y vértices 6 al 15 (o 46 al 55)

Se demuestra en Geometría, es

$$\boxed{r_1} = \frac{\sqrt{5}+1}{2} l = 1,61803399... l$$

Este mismo valor se puede deducir de los ya calculados anteriormente, considerando que " $r_1$ " es un cateto de un triángulo rectángulo de hipotenusa " $a$ " y el otro cateto " $\frac{f_1}{2}$ ". Su valor será:

$$\boxed{r_1} = \sqrt{a^2 - \left(\frac{f_1}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{11+2\sqrt{5}}}{2} l\right)^2 - \left[\frac{3\sqrt{\frac{5+2\sqrt{5}}{5}} - 2\sqrt{\frac{5-\sqrt{5}}{10}}}{2} \times l\right]^2} =$$

$$= \sqrt{\frac{11+4\sqrt{5}}{4} - \frac{9 \times \frac{5+2\sqrt{5}}{5} + 4 \times \frac{5-\sqrt{5}}{10} - 12 \sqrt{\frac{5+2\sqrt{5}}{5} \times \frac{5-\sqrt{5}}{10}}}{4}} \times l =$$

$$= \sqrt{\frac{11+4\sqrt{5}}{4} - \left(\frac{45+18\sqrt{5}}{20} + \frac{5-\sqrt{5}}{10} - 3 \sqrt{\frac{(5+2\sqrt{5})(5-\sqrt{5})}{50}}\right)} \times l =$$

No. 	Date 	Page 
To the Hon'ble Member of the Legislative Assembly 		
Subject: 		
Reference: 		
The undersigned has the honor to acknowledge the receipt of your letter of the 1st inst. in relation to the above subject and in reply to inform you that the same has been forwarded to the appropriate authorities for their consideration. 		
I am, Sir, very respectfully, 		
Yours faithfully, 		
(Signature) 		
(Name and Designation) 		
(Address) 		
(Telephone Number) 		
(Post Office Box) 		
(City) 		
(State) 		
(Country) 		



$$\begin{aligned}
 &= \sqrt{\frac{11 + 4\sqrt{5}}{4} - \frac{45 + 18\sqrt{5}}{20} - \frac{5 - \sqrt{5}}{10} + \frac{3}{5} \sqrt{\frac{25 + 10\sqrt{5} - 5\sqrt{5} - 10}{2}}} \times l = \\
 &= \sqrt{\frac{55 + 20\sqrt{5}}{20} - \frac{45 + 18\sqrt{5}}{20} - \frac{10 - 2\sqrt{5}}{20} + \frac{3}{5} \sqrt{\frac{15 + 5\sqrt{5}}{2}}} \times l = \\
 &= \sqrt{\frac{55 + 20\sqrt{5} - 45 - 18\sqrt{5} - 10 + 2\sqrt{5}}{20} + \frac{3}{5} \times \frac{\sqrt{5(3 + \sqrt{5})}}{\sqrt{2}}} \times l = \\
 &= \sqrt{\frac{4\sqrt{5}}{20} + \frac{3}{5} \times \sqrt{\frac{5}{2}} \times \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right)} \times l = \sqrt{\frac{\sqrt{5}}{5} + \frac{3}{5} \times \left(\frac{5}{2} + \frac{\sqrt{5}}{2}\right)} \times l = \\
 &= \sqrt{\frac{\sqrt{5}}{5} + \frac{3}{2} + \frac{3\sqrt{5}}{10}} \times l = \sqrt{\frac{2\sqrt{5} + 15 + 3\sqrt{5}}{10}} \times l = \sqrt{\frac{5\sqrt{5} + 15}{10}} \times l = \\
 &= \sqrt{\frac{\sqrt{5} + 3}{2}} l = \frac{\sqrt{3 + \sqrt{5}}}{\sqrt{2}} l = \frac{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}}{\sqrt{2}} \times l = \boxed{\frac{\sqrt{5} + 1}{2} \times l}
 \end{aligned}$$

valor coincidente con el del radio de la circunferencia circunscrita al decágono regular de lado "l" que, como indicamos al principio, se demuestra en Geometría.

Para el caso del dibujo, será:  $r_1 = 1,61\ 80\ 34... \times 24,63 = 39,9\ \text{mm.}$

Distancia "g<sub>2</sub>" de los vértices 16 al 25 al plano de la cara pentagonal 1 al 5, y de los vértices 36 al 45 a la cara pentagonal 56 al 60

Refiriéndonos a la lámina 38, vemos que la cara pentagonal 12-13-24-30-23, contigua a la cuadrada 3-6-13-12, están proyectadas ambas sobre I, según líneas rectas,

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \frac{1}{x}$ . It is shown that  $f(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$ . The second part of the paper is devoted to the study of the properties of the function  $g(x)$  defined by the equation  $g(x) = \frac{1}{x^2}$ . It is shown that  $g(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$ .

The third part of the paper is devoted to the study of the properties of the function  $h(x)$  defined by the equation  $h(x) = \frac{1}{x^3}$ . It is shown that  $h(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$ . The fourth part of the paper is devoted to the study of the properties of the function  $k(x)$  defined by the equation  $k(x) = \frac{1}{x^4}$ . It is shown that  $k(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$ .

$$\left( 3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5-\sqrt{5}}{10}} \right) l = \sqrt{\left( 3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5-\sqrt{5}}{10}} \right)^2} \times l =$$

$$= \sqrt{9 \times \frac{5+2\sqrt{5}}{5} + 4 \times \frac{5-\sqrt{5}}{10} - 12 \sqrt{\frac{5+2\sqrt{5}}{5} \times \frac{5-\sqrt{5}}{10}}} \times l =$$

$$= \sqrt{\frac{45+18\sqrt{5}}{5} + \frac{10-2\sqrt{5}}{5} - 12 \sqrt{\frac{25+10\sqrt{5}-5\sqrt{5}-10}{50}}} \times l =$$

$$= \sqrt{\frac{45+18\sqrt{5}+10-2\sqrt{5}}{5} - 12 \sqrt{\frac{15+5\sqrt{5}}{50}}} \times l = \sqrt{\frac{55+16\sqrt{5}}{5} - 12 \sqrt{\frac{3+\sqrt{5}}{10}}} \times l =$$

$$= \sqrt{\frac{55+16\sqrt{5}}{5} - \frac{12}{\sqrt{10}} \times \sqrt{3+\sqrt{5}}} \times l = \sqrt{\frac{55+16\sqrt{5}}{5} - \frac{12}{\sqrt{10}} \times \left( \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \right)} \times l =$$

$$= \sqrt{\frac{55+16\sqrt{5}}{5} - 12 \times \sqrt{\frac{5}{20}} - 12 \sqrt{\frac{1}{20}}} \times l = \sqrt{\frac{55+16\sqrt{5}}{5} - \frac{12}{2} - \frac{12}{2\sqrt{5}}} \times l =$$

$$= \sqrt{\frac{55+16\sqrt{5}}{5} - 6 - \frac{6\sqrt{5}}{5}} \times l = \sqrt{\frac{55+16\sqrt{5}-30-6\sqrt{5}}{5}} \times l =$$

$$= \sqrt{\frac{25+10\sqrt{5}}{5}} \times l = \boxed{\sqrt{5+2\sqrt{5}}} \times l = 3.07768353... l$$



por ser sus respectivos planos perpendiculares a I, por lo que la arista común 12-13, intersección de dichas caras, también es perpendicular a I.

En la figura 3 representamos el contorno del arquimedianos en dicha zona, que incluye el representado

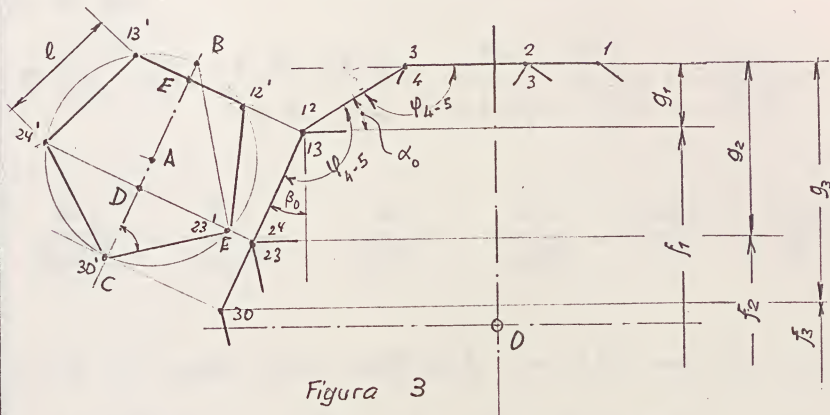


Figura 3

en la figura 2. La cara cuadrada 3-4-13-12, tiene contiguas las caras pentagonales 1 al 5 por la parte superior, y 12-13-24-30-23 por la inferior; esta última la hemos representado también abatida sobre el plano del dibujo (parte izquierda de la figura).

De la figura se deduce:

$$\varphi_{4.5} - \alpha_0 = \frac{\pi}{2} + \beta_0 \quad [1]$$

siendo " $\beta_0$ " el ángulo de proyección sobre III de la cara pentagonal 12-13-24-30-23.

De la [1] se deduce:



... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...



... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...

... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...  
 ... the ... of the ...

$$\frac{1}{\sqrt{5}} (\varphi_{4.5} - \alpha_0) = \frac{1}{\sqrt{2}} \left( \frac{\pi}{2} + \beta_0 \right) = -\frac{1}{\sqrt{2}} \beta_0 \quad [2]$$

pero ya hemos deducido en el cálculo de "g<sub>1</sub>" que

$$\frac{1}{\sqrt{2}} \varphi_{4.5} = -\frac{\sqrt{5}-1}{2} \quad \text{y} \quad \frac{1}{\sqrt{2}} \alpha_0 = \frac{\sqrt{5}-1}{2}$$

por lo que

$$\frac{1}{\sqrt{2}} (\varphi_{4.5} - \alpha_0) = \frac{\frac{1}{\sqrt{2}} \varphi_{4.5} - \frac{1}{\sqrt{2}} \alpha_0}{1 + \frac{1}{\sqrt{2}} \varphi_{4.5} - \frac{1}{\sqrt{2}} \alpha_0} = \frac{-\frac{\sqrt{5}-1}{2} - \frac{\sqrt{5}-1}{2}}{1 + \left(-\frac{\sqrt{5}-1}{2} + \frac{\sqrt{5}-1}{2}\right)} = \frac{-(\sqrt{5}-1)}{1 - \left(\frac{\sqrt{5}-1}{2}\right)^2} =$$

$$= \frac{\sqrt{5}-1}{\left(\frac{\sqrt{5}-1}{2}\right)^2 - 1} = \frac{\sqrt{5}-1}{\frac{5+1-2\sqrt{5}}{4} - 1} = \frac{\sqrt{5}-1}{\frac{3-\sqrt{5}}{2} - 1} = \frac{\sqrt{5}-1}{\frac{1-\sqrt{5}}{2}} = \frac{2(\sqrt{5}-1)}{1-\sqrt{5}} = -\frac{2(\sqrt{5}-1)}{\sqrt{5}-1} =$$

$= -2$  valor que sustituido en [2] nos da

$$-2 = -\frac{1}{\sqrt{2}} \beta_0 \quad \text{y} \quad \frac{1}{\sqrt{2}} \beta_0 = 2 \quad \text{y de aquí}$$

$$\frac{1}{\sqrt{2}} \beta_0 = \frac{1}{2}$$

[3]

de esta última se deduce:

$$\cos \beta_0 = \frac{1}{\sqrt{1 + \frac{1}{4} \beta_0^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{1 + \frac{1}{4}}} = \frac{1}{\sqrt{\frac{5}{4}}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad [4]$$



Por otra parte, en la cara pentagonal 12'-13'-24'-30'-23', situada en la parte izquierda de la fig. 3, tendremos:

$\overline{CE}^2 = \overline{CD} \times \overline{CB}$  (el triángulo C-E-B es rectángulo, inscrito en la circunferencia de radio C-A); de donde

Main body of handwritten text, appearing to be a letter or document. The text is very faint and mostly illegible.

$$\overline{CD} \times \overline{CB} = \overline{CE}^2 \quad \text{"} \quad \boxed{\overline{CD}} = \frac{\overline{CE}^2}{\overline{CB}} = \frac{l^2}{2 d_5} = \frac{l^2}{2 \sqrt{\frac{5+\sqrt{5}}{10}} \times l} =$$

$$= \frac{1}{\sqrt{\frac{4 \times (5+\sqrt{5})}{10}}} \times l = \frac{1}{\sqrt{\frac{2(5+\sqrt{5})}{5}}} \times l = \sqrt{\frac{5}{2 \times (5+\sqrt{5})}} \times l = \sqrt{\frac{5(5-\sqrt{5})}{2 \times 20}} \times l =$$

$$= \boxed{\sqrt{\frac{5-\sqrt{5}}{8}} \times l}$$

Pero por otra parte tenemos:

$$\boxed{\overline{DE}} = \overline{CE} - \overline{CD} = \overline{CA} + \overline{AE} - \overline{CD} = d_5 + k - \overline{CD} = \sqrt{\frac{5+\sqrt{5}}{10}} \times l + \sqrt{\frac{5+2\sqrt{5}}{20}} \times l -$$

$$- \sqrt{\frac{5-\sqrt{5}}{8}} \times l = \left[ \left( \sqrt{\frac{5+\sqrt{5}}{10}} - \sqrt{\frac{5-\sqrt{5}}{8}} \right) + \sqrt{\frac{5+2\sqrt{5}}{20}} \right] \times l =$$

$$= \left[ \left( \sqrt{\frac{5+\sqrt{5}}{10}} - \sqrt{\frac{5-\sqrt{5}}{8}} \right)^2 + \sqrt{\frac{5+2\sqrt{5}}{20}} \right] \times l = \left[ \sqrt{\frac{5+\sqrt{5}}{10}} + \frac{5-\sqrt{5}}{8} - 2 \times \sqrt{\frac{5+\sqrt{5}}{10} \times \frac{5-\sqrt{5}}{8}} \right] +$$

$$+ \sqrt{\frac{5+2\sqrt{5}}{20}} \times l = \left[ \sqrt{\frac{20+4\sqrt{5}+25-5\sqrt{5}}{40}} - 2 \times \sqrt{\frac{20}{80}} + \sqrt{\frac{5+2\sqrt{5}}{20}} \right] \times l =$$

$$= \left[ \sqrt{\frac{45-\sqrt{5}}{40}} - 1 + \sqrt{\frac{5+2\sqrt{5}}{20}} \right] \times l = \left[ \sqrt{\frac{5-\sqrt{5}}{40}} + \sqrt{\frac{5+2\sqrt{5}}{20}} \right] \times l =$$

$$= \sqrt{\left( \sqrt{\frac{5-\sqrt{5}}{40}} + \sqrt{\frac{5+2\sqrt{5}}{20}} \right)^2} \times l = \sqrt{\frac{5-\sqrt{5}}{40} + \frac{5+2\sqrt{5}}{20} + 2 \times \sqrt{\frac{(5-\sqrt{5})(5+2\sqrt{5})}{40 \times 20}}} \times l =$$

$$= \sqrt{\frac{5-\sqrt{5}+10+4\sqrt{5}}{40} + \sqrt{\frac{25-5\sqrt{5}+10\sqrt{5}-10}{10 \times 20}}} \times l = \sqrt{\frac{15+3\sqrt{5}}{40} + \sqrt{\frac{15+5\sqrt{5}}{200}}} \times l =$$

$$= \sqrt{\frac{15+3\sqrt{5}}{40} + \sqrt{\frac{3+\sqrt{5}}{40}}} \times l = \sqrt{\frac{15+3\sqrt{5}}{40} + \frac{\sqrt{3+\sqrt{5}}}{2\sqrt{10}}} \times l = \sqrt{\frac{15+3\sqrt{5}}{40} + \frac{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}}{2\sqrt{10}}} \times l$$

$$= \sqrt{\frac{15+3\sqrt{5}}{40} + \frac{\sqrt{\frac{50}{2}} + \sqrt{\frac{10}{2}}}{20}} \times l = \sqrt{\frac{15+3\sqrt{5}}{40} + \frac{5+\sqrt{5}}{20}} \times l = \sqrt{\frac{15+3\sqrt{5}+10+2\sqrt{5}}{40}} \times l =$$

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$$= \sqrt{\frac{25 + 5\sqrt{5}}{40}} \cdot l = \boxed{\sqrt{\frac{5 + \sqrt{5}}{8}} \cdot l}$$

Finalmente de la figura 3, se deduce que

$$\boxed{g_2} = \overline{DE} \cos \beta_0 + g_1 = \sqrt{\frac{5 + \sqrt{5}}{8}} \cdot l \cdot \frac{2\sqrt{5}}{5} + \sqrt{\frac{5 - \sqrt{5}}{10}} \cdot l =$$

$$= \left( \frac{2}{5} \sqrt{\frac{5 + \sqrt{5}}{8}} \cdot 5 + \sqrt{\frac{5 - \sqrt{5}}{10}} \right) \cdot l = \left( \frac{1}{5} \sqrt{\frac{5(5 + \sqrt{5})}{2}} + \sqrt{\frac{5 - \sqrt{5}}{10}} \right) \cdot l =$$

$$= \left( \sqrt{\frac{5 + \sqrt{5}}{10}} + \sqrt{\frac{5 - \sqrt{5}}{10}} \right) l = \sqrt{\left( \sqrt{\frac{5 + \sqrt{5}}{10}} + \sqrt{\frac{5 - \sqrt{5}}{10}} \right)^2} \cdot l =$$

$$= \sqrt{\frac{5 + \sqrt{5}}{10} + \frac{5 - \sqrt{5}}{10} + 2 \cdot \sqrt{\frac{5 + \sqrt{5}}{10}} \cdot \sqrt{\frac{5 - \sqrt{5}}{10}}} \cdot l = \sqrt{\frac{10}{10} + 2 \sqrt{\frac{20}{100}}} \cdot l =$$

$$= \sqrt{1 + \sqrt{\frac{80}{100}}} \cdot l = \sqrt{1 + \sqrt{\frac{4}{5}}} \cdot l = \sqrt{1 + \frac{2}{\sqrt{5}}} \cdot l = \sqrt{1 + \frac{2\sqrt{5}}{5}} \cdot l =$$

$$= \boxed{\sqrt{\frac{5 + 2\sqrt{5}}{5}}} \cdot l = 1,3763819\dots \cdot l$$

Para el caso del dibujo, será:  $g_2 = 1,3763819\dots \times 24,63 = 33,9$  mm

Distancia "f" entre los dos planos paralelos a II, que contienen los vértices 16 al 25 y 36 al 45, respectivamente

Se obtiene por diferencias de las alturas "c<sub>5</sub>" y "g<sub>2</sub>", ya calculadas.

$$\boxed{f_2} = 2(c_5 - g_2) = 2 \cdot \left( \frac{3}{2} \sqrt{\frac{5 + 2\sqrt{5}}{5}} \cdot l - \sqrt{\frac{5 + 2\sqrt{5}}{5}} \cdot l \right) =$$

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \frac{1}{x}$ . It is shown that  $f(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a horizontal asymptote at  $y = 0$  and a vertical asymptote at  $x = 0$ .

In the second part, we consider the function  $g(x) = \ln x$  and its properties. It is shown that  $g(x)$  is an increasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

The third part of the paper is devoted to the study of the function  $h(x) = e^x$  and its properties. It is shown that  $h(x)$  is an increasing function on the interval  $(-\infty, \infty)$  and that it has a horizontal asymptote at  $y = 0$ .

In the fourth part, we consider the function  $k(x) = \frac{1}{e^x}$  and its properties. It is shown that  $k(x)$  is a decreasing function on the interval  $(-\infty, \infty)$  and that it has a horizontal asymptote at  $y = 0$ .

The fifth part of the paper is devoted to the study of the function  $l(x) = \frac{1}{x^2}$  and its properties. It is shown that  $l(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

In the sixth part, we consider the function  $m(x) = \frac{1}{x^3}$  and its properties. It is shown that  $m(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

The seventh part of the paper is devoted to the study of the function  $n(x) = \frac{1}{x^4}$  and its properties. It is shown that  $n(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

In the eighth part, we consider the function  $o(x) = \frac{1}{x^5}$  and its properties. It is shown that  $o(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

The ninth part of the paper is devoted to the study of the function  $p(x) = \frac{1}{x^6}$  and its properties. It is shown that  $p(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

In the tenth part, we consider the function  $q(x) = \frac{1}{x^7}$  and its properties. It is shown that  $q(x)$  is a decreasing function on the interval  $(0, \infty)$  and that it has a vertical asymptote at  $x = 0$ .

$$= \left( 3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5+2\sqrt{5}}{5}} \right) l = \boxed{\sqrt{\frac{5+2\sqrt{5}}{5}}} l = g_2 = 1,3763819... l$$

El cálculo anterior nos demuestra que

$$\boxed{g_2 = f_2}$$

Radio "r<sub>2</sub>" de la circunferencia circunscrita al polígono que tiene por vértices 16 al 25 y también a los 36 al 45.

Dicho radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto es " $\frac{f_2}{2}$ " (ver lám. 38), de valores ya calculados.

$$\boxed{r_2} = \sqrt{a^2 - \left(\frac{f_2}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{11+4\sqrt{5}}}{2} l\right)^2 - \left(\frac{1}{2} \sqrt{\frac{5+2\sqrt{5}}{5}} l\right)^2} =$$

$$= \sqrt{\frac{11+4\sqrt{5}}{4} - \frac{1}{4} \times \frac{5+2\sqrt{5}}{5}} l = \sqrt{\frac{55+20\sqrt{5}-5-2\sqrt{5}}{20}} l = \sqrt{\frac{50+18\sqrt{5}}{20}} l =$$

$$= \boxed{\sqrt{\frac{25+9\sqrt{5}}{10}}} l = 2,12435544... l$$

Para el caso del dibujo, será:  $r_2 = 2,12435544 \times 24,63 = 52,31 \text{ mm}$

Distancia "g<sub>3</sub>" de los vértices 26 al 30 al plano de la cara pentagonal 1 al 5, y de los vértices 31 al 35 a la cara pentagonal 56 al 60.

En la determinación de los valores "g<sub>3</sub>", "f<sub>3</sub>" y "r<sub>3</sub>",

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is well known that this function is increasing and concave down on the interval  $(-\infty, \infty)$ .

In the second part, we consider the function  $g(x)$  defined by the equation

$$g(x) = \int_0^x \frac{t}{1+t^2} dt$$

This function is also increasing and concave down on the interval  $(-\infty, \infty)$ .

The third part of the paper is devoted to the study of the function  $h(x)$  defined by the equation

$$h(x) = \int_0^x \frac{t^2}{1+t^2} dt$$

This function is also increasing and concave down on the interval  $(-\infty, \infty)$ .

In the fourth part, we consider the function  $k(x)$  defined by the equation

$$k(x) = \int_0^x \frac{t^3}{1+t^2} dt$$

This function is also increasing and concave down on the interval  $(-\infty, \infty)$ .

requerimos el mismo proceso que para los correspondientes " $g_2$ ", " $f_2$ " y " $h_2$ ", ya calculados, y con las mismas referencias a la figura 3. De esta se deduce que:

$$\begin{aligned}
 [g_3] &= \overline{CF} \cos \beta_0 + g_1 = (d_5 + k) \cos \beta_0 + g_1 = \\
 &= \left[ \sqrt{\frac{5+\sqrt{5}}{10}} \ell + \sqrt{\frac{5+2\sqrt{5}}{20}} \ell \right] \times \frac{2\sqrt{5}}{5} + \sqrt{\frac{5-\sqrt{5}}{10}} \ell = \left( \frac{2}{5} \sqrt{\frac{5+\sqrt{5}}{10}} \times 5 + \right. \\
 &+ \frac{2}{5} \sqrt{\frac{5+2\sqrt{5}}{20}} \times 5 + \left. \sqrt{\frac{5-\sqrt{5}}{10}} \right) \times \ell = \left( \frac{2}{5} \sqrt{\frac{5+\sqrt{5}}{2}} + \frac{2}{5} \sqrt{\frac{5+2\sqrt{5}}{4}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right) \ell = \\
 &= \left( \frac{1}{5} \sqrt{2(5+\sqrt{5})} + \frac{1}{5} \sqrt{5+2\sqrt{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right) \ell = \left( \frac{\sqrt{10+2\sqrt{5}}}{5} + \frac{\sqrt{5+2\sqrt{5}}}{5} + \sqrt{\frac{5-\sqrt{5}}{10}} \right) \ell = \\
 &= \left[ \frac{1}{5} \times \sqrt{(\sqrt{10+2\sqrt{5}}) + \sqrt{5+2\sqrt{5}}}^2 + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \times \ell = \left[ \frac{1}{5} \sqrt{10+2\sqrt{5} + 5+2\sqrt{5}} + 2\sqrt{(10+2\sqrt{5})(5+2\sqrt{5})} \right. \\
 &+ \left. \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 2\sqrt{50+10\sqrt{5}} + 20\sqrt{5} + 20 + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \\
 &= \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 2\sqrt{70+30\sqrt{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 2\sqrt{10} \times \sqrt{7+3\sqrt{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \\
 &= \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 2\sqrt{10} \times \left( \sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}} \right) + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \times \ell = \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 2\sqrt{\frac{90}{2}} + 2\sqrt{\frac{50}{2}} + \right. \\
 &+ \left. \sqrt{\frac{5-\sqrt{5}}{10}} \right] \times \ell = \left[ \frac{1}{5} \sqrt{15+4\sqrt{5}} + 6\sqrt{5} + 10 + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \times \ell = \left[ \frac{1}{5} \sqrt{35+10\sqrt{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \\
 &= \left[ \sqrt{\frac{5+2\sqrt{5}}{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right] \ell = \sqrt{\left( \sqrt{\frac{5+2\sqrt{5}}{5}} + \sqrt{\frac{5-\sqrt{5}}{10}} \right)^2} \times \ell = \\
 &= \sqrt{\frac{5+2\sqrt{5}}{5} + \frac{5-\sqrt{5}}{10} + 2\sqrt{\frac{5+2\sqrt{5}}{5} \times \frac{5-\sqrt{5}}{10}}} \times \ell = \sqrt{\frac{10+4\sqrt{5}+5-\sqrt{5}}{10} + 2\sqrt{\frac{25+10\sqrt{5}-5\sqrt{5}-10}{50}}} \times \ell =
 \end{aligned}$$



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$$= \sqrt{\frac{15+3\sqrt{5}}{10}} + 2\sqrt{\frac{15+5\sqrt{5}}{50}} \times l = \sqrt{\frac{15+3\sqrt{5}}{10}} + \sqrt{\frac{60+20\sqrt{5}}{50}} \times l =$$

$$= \sqrt{\frac{15+3\sqrt{5}}{10}} + \sqrt{\frac{6+2\sqrt{5}}{5}} \times l = \sqrt{\frac{15+3\sqrt{5}}{10}} + \sqrt{\frac{2}{5}} \times \sqrt{3+\sqrt{5}} \times l =$$

$$= \sqrt{\frac{15+3\sqrt{5}}{10}} + \sqrt{\frac{2}{5}} \times \left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right) \times l = \sqrt{\frac{15+3\sqrt{5}}{10}} + \sqrt{\frac{10}{10}} + \sqrt{\frac{2}{10}} \times l =$$

$$= \sqrt{\frac{15+3\sqrt{5}}{10}} + 1 + \frac{\sqrt{5}}{5} \times l = \sqrt{\frac{15+3\sqrt{5}+10+2\sqrt{5}}{10}} \times l = \sqrt{\frac{25+5\sqrt{5}}{10}} \times l = \boxed{\sqrt{\frac{5+\sqrt{5}}{2}}} \times l =$$

$$= 1.90211408... \times l$$

Para el caso del dibujo, será:  $g_3 = 1.90211408... \times 24.63 = 46.8 \text{ mm}$

Distancia "f<sub>3</sub>" entre los planos paralelos a II, que contienen los vértices 26 al 30 y 31 al 35 respectivamente.

Se obtiene por diferencias de las alturas "c<sub>5</sub>" y "g<sub>3</sub>", ya calculadas.

$$\boxed{f_3} = 2(c_5 - g_3) = 2 \times \left(\frac{3}{2} \sqrt{\frac{5+2\sqrt{5}}{5}} - \sqrt{\frac{5+\sqrt{5}}{2}}\right) \times l =$$

$$= \left(3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5+\sqrt{5}}{2}}\right) \times l = \sqrt{\left(3 \sqrt{\frac{5+2\sqrt{5}}{5}} - 2 \sqrt{\frac{5+\sqrt{5}}{2}}\right)^2} \times l =$$

$$= \sqrt{9 \times \frac{5+2\sqrt{5}}{5} + 4 \times \frac{5+\sqrt{5}}{2} - 12 \times \sqrt{\frac{5+2\sqrt{5}}{5}} \times \sqrt{\frac{5+\sqrt{5}}{2}}} \times l =$$

$$= \sqrt{\frac{45+18\sqrt{5}}{5} + 10+2\sqrt{5} - 12 \sqrt{\frac{25+10\sqrt{5}+5\sqrt{5}+10}{10}}} \times l = \sqrt{\frac{45+18\sqrt{5}+50+10\sqrt{5}}{5}} -$$

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined on the interval  $[0, 1]$ . It is shown that  $f(x)$  is continuous and differentiable on this interval. The derivative of  $f(x)$  is given by the formula  $f'(x) = \dots$ . The function  $f(x)$  is also shown to be concave down on  $[0, 1]$ .

2. In the second part, we consider the function  $g(x)$  defined on the interval  $[0, 1]$ . It is shown that  $g(x)$  is continuous and differentiable on this interval. The derivative of  $g(x)$  is given by the formula  $g'(x) = \dots$ . The function  $g(x)$  is also shown to be concave up on  $[0, 1]$ .

3. The third part of the paper is devoted to the study of the function  $h(x)$  defined on the interval  $[0, 1]$ . It is shown that  $h(x)$  is continuous and differentiable on this interval. The derivative of  $h(x)$  is given by the formula  $h'(x) = \dots$ . The function  $h(x)$  is also shown to be concave down on  $[0, 1]$ .

4. In the fourth part, we consider the function  $k(x)$  defined on the interval  $[0, 1]$ . It is shown that  $k(x)$  is continuous and differentiable on this interval. The derivative of  $k(x)$  is given by the formula  $k'(x) = \dots$ . The function  $k(x)$  is also shown to be concave up on  $[0, 1]$ .

5. The fifth part of the paper is devoted to the study of the function  $l(x)$  defined on the interval  $[0, 1]$ . It is shown that  $l(x)$  is continuous and differentiable on this interval. The derivative of  $l(x)$  is given by the formula  $l'(x) = \dots$ . The function  $l(x)$  is also shown to be concave down on  $[0, 1]$ .

6. In the sixth part, we consider the function  $m(x)$  defined on the interval  $[0, 1]$ . It is shown that  $m(x)$  is continuous and differentiable on this interval. The derivative of  $m(x)$  is given by the formula  $m'(x) = \dots$ . The function  $m(x)$  is also shown to be concave up on  $[0, 1]$ .

7. The seventh part of the paper is devoted to the study of the function  $n(x)$  defined on the interval  $[0, 1]$ . It is shown that  $n(x)$  is continuous and differentiable on this interval. The derivative of  $n(x)$  is given by the formula  $n'(x) = \dots$ . The function  $n(x)$  is also shown to be concave down on  $[0, 1]$ .

8. In the eighth part, we consider the function  $o(x)$  defined on the interval  $[0, 1]$ . It is shown that  $o(x)$  is continuous and differentiable on this interval. The derivative of  $o(x)$  is given by the formula  $o'(x) = \dots$ . The function  $o(x)$  is also shown to be concave up on  $[0, 1]$ .

9. The ninth part of the paper is devoted to the study of the function  $p(x)$  defined on the interval  $[0, 1]$ . It is shown that  $p(x)$  is continuous and differentiable on this interval. The derivative of  $p(x)$  is given by the formula  $p'(x) = \dots$ . The function  $p(x)$  is also shown to be concave down on  $[0, 1]$ .

10. In the tenth part, we consider the function  $q(x)$  defined on the interval  $[0, 1]$ . It is shown that  $q(x)$  is continuous and differentiable on this interval. The derivative of  $q(x)$  is given by the formula  $q'(x) = \dots$ . The function  $q(x)$  is also shown to be concave up on  $[0, 1]$ .

$$\begin{aligned}
 & -12 \sqrt{\frac{35+15\sqrt{5}}{10}} \times l = \sqrt{\frac{95+28\sqrt{5}}{5}} - 12 \sqrt{\frac{7+3\sqrt{5}}{2}} \times l = \\
 & = \sqrt{\frac{95+28\sqrt{5}}{5}} - \frac{12}{\sqrt{2}} \times \sqrt{7+3\sqrt{5}} \times l = \sqrt{\frac{95+28\sqrt{5}}{5}} - \frac{12}{\sqrt{2}} \left( \sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}} \right) \times l = \\
 & = \sqrt{\frac{95+28\sqrt{5}}{5}} - 12 \times \left( \sqrt{\frac{9}{4}} + \sqrt{\frac{5}{4}} \right) \times l = \sqrt{\frac{95+28\sqrt{5}}{5}} - 12 \times \left( \frac{3}{2} + \frac{\sqrt{5}}{2} \right) \times l = \\
 & = \sqrt{\frac{95+28\sqrt{5}}{5}} - 18 - 6\sqrt{5} \times l = \sqrt{\frac{95+28\sqrt{5}}{5}} - 90 - 30\sqrt{5} \times l = \boxed{\sqrt{\frac{5-2\sqrt{5}}{5}} \times l} = \\
 & = 0,32 \ 49 \ 19 \ 6... \ l
 \end{aligned}$$

Para el caso del dibujo, será:  $f_3 = 0,32 \ 49 \ 19 \ 6... \times 24,63 = 8,0 \text{ mm}$

Radio "f<sub>3</sub>" de la circunferencia circunscrita al polígono que tiene por vértices los 26 al 35.

Dicho radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto " $\frac{f_3}{2}$ " (ver lám. 38), de valores ya calculados.

$$\begin{aligned}
 \boxed{f_3} &= \sqrt{a^2 - \left(\frac{f_3}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{11+4\sqrt{5}}}{2} l\right)^2 - \left(\frac{\sqrt{5-2\sqrt{5}}}{5} l : 2\right)^2} = \\
 &= \sqrt{\frac{11+4\sqrt{5}}{4} - \frac{5-2\sqrt{5}}{5} : 4} \times l = \sqrt{\frac{11+4\sqrt{5}}{4} - \frac{5-2\sqrt{5}}{20}} l = \\
 &= \sqrt{\frac{55+20\sqrt{5}-5+2\sqrt{5}}{20}} l = \sqrt{\frac{50+22\sqrt{5}}{20}} \times l = \boxed{\sqrt{\frac{25+11\sqrt{5}}{10}} l} = 2,22 \ 70 \ 32 \ 73... l
 \end{aligned}$$

Para el caso del dibujo, será:  $f_3 = 2,22 \ 70 \ 32 \ 73... \times 24,63 = 54,9 \text{ mm}$

Handwritten text in Devanagari script, likely a manuscript or letter. The text is arranged in approximately 12 horizontal lines. The script is cursive and somewhat faded. The first line appears to be a header or title, possibly starting with 'श्री' (Shri). The subsequent lines contain several lines of text, some of which are enclosed in brackets or boxes, suggesting a list or a structured document. The text is written on aged, slightly discolored paper.



En la proyección II de los vértices 26 al 35 se puede considerar que prácticamente están sobre la proyección de la esfera circumsrita al arquimedianos. La diferencia de radios, en el ejemplo de la lámina es tan sólo de 0,1 mm.

En el cuadro sinóptico que damos a continuación, resumimos los resultados de los valores complementarios deducidos.

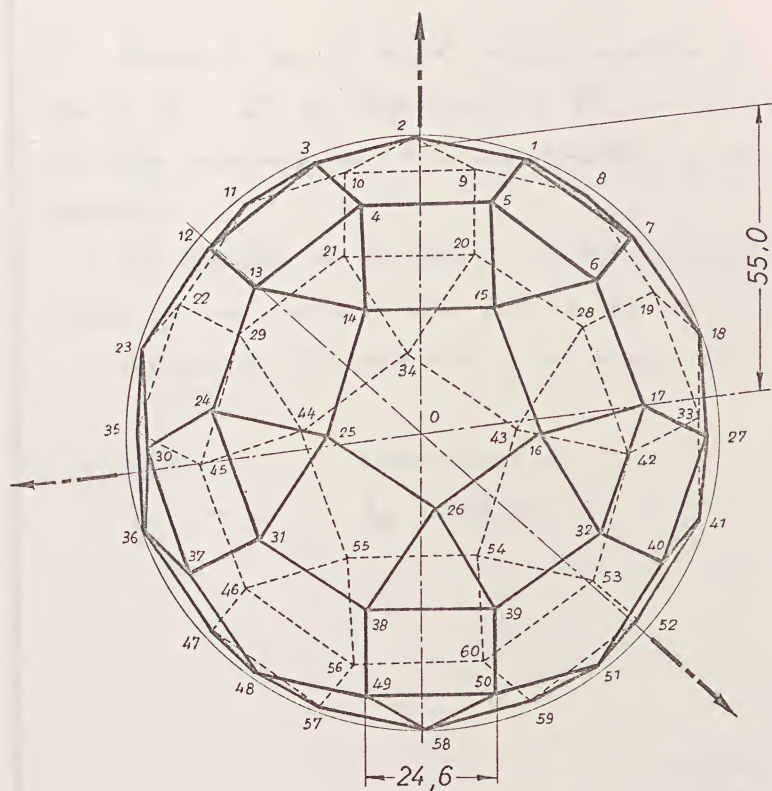
CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
$n$	$\frac{\sqrt{3}}{2} l$	0,86 60 25.... $l$
$k$	$\sqrt{\frac{5+2\sqrt{5}}{20}} l$	0,68 81 91.... $l$
$f_1$	$\sqrt{5+2\sqrt{5}} l$	3,07 76 84.... $l$
$f_2$	$\sqrt{\frac{5+2\sqrt{5}}{5}} l$	1,37 63 82.... $l$
$f_3$	$\sqrt{\frac{5-2\sqrt{5}}{5}} l$	0,32 49 20... $l$
$g_1$	$\sqrt{\frac{5-\sqrt{5}}{10}} l$	0,52 57 31.... $l$
$g_2$	$\sqrt{\frac{5+2\sqrt{5}}{5}} l$	1,37 63 82.... $l$
$g_3$	$\sqrt{\frac{5+\sqrt{5}}{2}} l$	1,90 21 14.... $l$
$r_1$	$\frac{\sqrt{5+1}}{2} l$	1,61 80 34.... $l$
$r_2$	$\sqrt{\frac{25+9\sqrt{5}}{10}} l$	2,12 42 55... $l$
$r_3$	$\sqrt{\frac{25+11\sqrt{5}}{10}} l$	2,22 70 33... $l$
$f_2 = g_2 = 2k = \frac{2G_5}{3}$ (relaciones notables)		

The following table shows the results of the experiment conducted on the 10th of May 1900. The results are given in the form of a table, and the data is as follows:

Table showing the results of the experiment conducted on the 10th of May 1900.

Time (min)	Temperature (°C)	Pressure (mm Hg)
0	20.0	760.0
10	21.5	755.0
20	23.0	750.0
30	24.5	745.0
40	26.0	740.0
50	27.5	735.0
60	29.0	730.0
70	30.5	725.0
80	32.0	720.0
90	33.5	715.0
100	35.0	710.0
110	36.5	705.0
120	38.0	700.0
130	39.5	695.0
140	41.0	690.0
150	42.5	685.0
160	44.0	680.0
170	45.5	675.0
180	47.0	670.0
190	48.5	665.0
200	50.0	660.0



Arquimediato VI



ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano VII, en el que en cada vértice concurren un triángulo equilátero y dos hexágonos regulares.

La longitud de su lado es de 46.9 mm, y las coordenadas de su centro O, son (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

DATOS

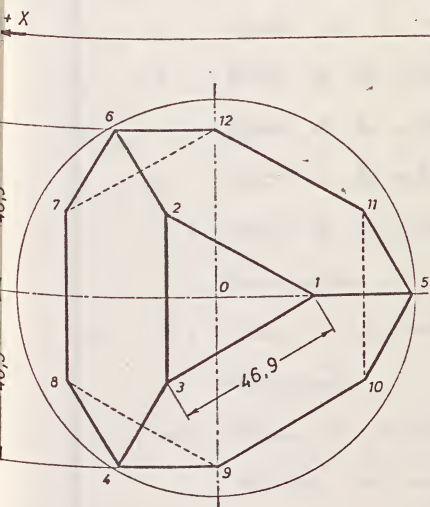
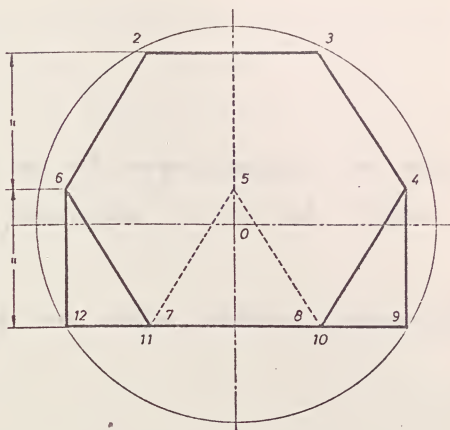
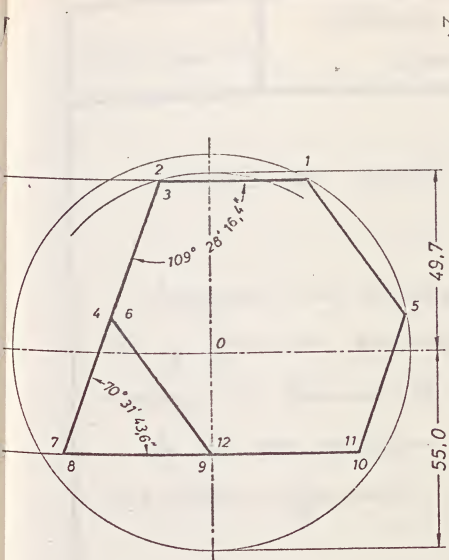
O (72, 72, 85) mm.

$l_{VII} = 46.9 \text{ mm}$









## ARQUIMEDIANO VII

Número de caras triangulares.....	$C_3 = 4$
Número de caras exagonales.....	$C_6 = 4$
Número de vértices.....	$V = 12$
Número de aristas.....	$A = 18$
Número de caras de un ángulo sólido.....	$1P_3 + 2P_6$

## ENUNCIADO

Representar por el método gráfico-co-analítico en los planos I, II y III, el Arquimiliano VII en el que en cada vértice concurren un triángulo equilátero y dos exágonos regulares.

La longitud de su lado es de 46,9 milímetros, y las coordenadas de su centro O, son: O(72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						
Alumno:						Curso
Escala						
1:1	Arquimiliano VII					Lámina 39
						Curso 19 - 19



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CONSIDERACIONES PREVIAS

Requiere, en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el del "Arquimedianos I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

- $l$  = Arista del Arquimedianos VII (dato del ejercicio).
- $a$  = Radio de la esfera circunscrita.
- $b$  = Radio de la esfera tangente a las aristas.
- $c_3$  = Radio de la esfera tangente a las caras triangulares.
- $c_6$  = Radio de la esfera tangente a las caras hexagonales.
- $d_3$  = Radio de la circunferencia circunscrita a una cara triangular.
- $d_6$  = Radio de la circunferencia circunscrita a una cara hexagonal.
- $m$  = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.
- $\alpha_3$  = Ángulo rectilíneo del diedro formado por una cara triangular con el plano diametral del arquimedianos, que pasa por una arista de aquella.
- $\alpha_6$  = Ángulo rectilíneo del diedro formado por una cara





exagonal con el plano diametral del arquimedeano que pasa por una arista de aquella.

$\varphi_{3-6}$  = Ángulo rectilíneo del diedro formado por una cara triangular y otra exagonal.

$\varphi_{6-6}$  = Ángulo rectilíneo del diedro formado por dos caras hexagonales.

$S$  = Superficie

$V$  = Volumen.

### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedeano, nos indica que se compone de 4 caras triangulares y 4 caras hexagonales; 12 vértices y 18 aristas.

En cada vértice concurren un triángulo y dos hexágonos, todos regulares.

Así pues, tendremos que

$$\text{ARQUIMEDEANO VII } (1 P_3 + 2 P_6); \quad C_3 = 4; \quad C_6 = 4; \quad V = 12; \quad A = 18$$

Cálculo de sus magnitudes

Arista "l" del arquimedeano

Dato del ejercicio

Received of Mr. J. H. ...  
the sum of ...  
for ...  
...

...

...

...

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido.

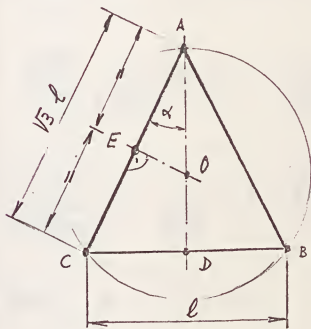


Figura 1

Dicho polígono (fig. 1) es un triángulo isósceles, cuya base BC es la arista "l" del arquimedianos (lado de la cara triangular), y sus otros dos lados iguales  $AC = AB$ , corresponden a la diagonal de la cara escagonal también de lado "l".

La geometría nos enseña que esta diagonal, es

$$AC = AB = \sqrt{3} l$$

De la figura se deduce:

$$\begin{aligned} \overline{AD} &= \sqrt{AC^2 - CD^2} = \sqrt{(\sqrt{3} l)^2 - \left(\frac{l}{2}\right)^2} = \sqrt{3l^2 - \frac{l^2}{4}} = \sqrt{3 - \frac{1}{4}} \times l = \\ &= \sqrt{\frac{11}{4}} \times l = \frac{\sqrt{11}}{2} l; \quad \text{por lo que será:} \end{aligned}$$

$$\cos \alpha = \frac{\overline{AD}}{AC} = \frac{\frac{\sqrt{11}}{2} l}{\sqrt{3} l} = \frac{\sqrt{11}}{2\sqrt{3}} = \frac{\sqrt{33}}{6} \quad \text{y también:}$$

$$\overline{AD} = \boxed{m} = \frac{\overline{AE}}{\cos \alpha} = \frac{\frac{\sqrt{3}}{2} l}{\frac{\sqrt{33}}{6}} = \frac{\sqrt{3}}{2} l \cdot \frac{6}{\sqrt{33}} = \frac{6\sqrt{3}}{2\sqrt{33}} l = 3\sqrt{\frac{3}{33}} l = 3\sqrt{\frac{1}{11}} l = 3 \times \frac{\sqrt{11}}{11} l$$

Para el caso del dibujo será:

$$= \frac{3\sqrt{11}}{11} l = 0,90453403 \dots l$$

$$m = 0,90453403 \times 46,7 = 42,4 \text{ mm}$$

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Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33) a este caso particular.

$$a = \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\frac{3\sqrt{11}}{11}l\right)^2}} = \frac{1}{2\sqrt{1 - \frac{9 \times 11}{11^2}}} l = \frac{1}{2\sqrt{1 - \frac{9}{11}}} l =$$

$$= \frac{1}{2\sqrt{\frac{2}{11}}} l = \frac{1}{\sqrt{\frac{8}{11}}} l = \sqrt{\frac{11}{8}} l = \frac{\sqrt{22}}{4} l = \boxed{1,17260394\dots l}$$

Para  $a = 55 \text{ mm}$ ,  $l = \frac{55}{1,17260394} = 46,904 \text{ mm}$ .

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3], (ver lám. 33), tendremos:

$$b = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{22}}{4}l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{22}{16} - \frac{1}{4}} l = \sqrt{\frac{11}{8} - \frac{1}{4}} l =$$

$$= \sqrt{\frac{11-2}{8}} l = \sqrt{\frac{9}{8}} l = \frac{3}{2\sqrt{2}} l = \boxed{\frac{3\sqrt{2}}{4} l} = 1,06066017\dots l$$

Para el caso del dibujo, será:  $b = 1,06066017\dots \times 46,904 = 49,7 \text{ mm}$

Radio "d<sub>3</sub>" de la circunferencia circunscrita a una cara triangular de lado "l"



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DEPARTMENT OF THE HISTORY OF ARTS  
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Se demuestra en Geometría, es

$$d_3 = \frac{\sqrt{3}}{3} l = 0,57735027... l$$

Para el caso del dibujo será:  $d_3 = 0,57735027... \times 46,9 = 27,1 \text{ mm.}$

Radio "d<sub>6</sub>" de la circunferencia circunscrita a una cara exagonal de lado "l"

Se demuestra en Geometría, es

$$d_6 = l$$

Radio "C<sub>3</sub>" de la esfera tangente a las caras triangulares de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$C_3 = \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\frac{\sqrt{3}}{2} l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{22}{16} - \frac{3}{9}} \times l =$$

$$= \sqrt{\frac{11}{8} - \frac{1}{3}} \times l = \sqrt{\frac{33-8}{24}} \times l = \sqrt{\frac{25}{24}} \times l = \frac{5}{2\sqrt{6}} \times l = \frac{5\sqrt{6}}{12} l =$$

$$= 1,02062073... l$$

Para el caso del dibujo, será:  $C_3 = 1,02062073 \times 46,9 = 47,9 \text{ mm.}$

Radio "C<sub>6</sub>" de la esfera tangente a las caras exagonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

Q.1. A particle is moving with a constant velocity of 10 m/s. Find the distance covered by it in 5 seconds.

$$v = 10 \text{ m/s}$$

Given:  $v = 10 \text{ m/s}$ ,  $t = 5 \text{ s}$

To find: Distance covered ( $s$ )

Sol: We know that  $v = \frac{s}{t}$

$\therefore s = vt$

$$s = 10 \times 5$$

$\therefore s = 50 \text{ m}$

∴ The distance covered by the particle is 50 m.

Q.2. A car starts from rest and accelerates uniformly to a speed of 20 m/s in 10 seconds. Find the acceleration.

$$u = 0 \text{ m/s}, v = 20 \text{ m/s}, t = 10 \text{ s}$$

$$v = u + at$$

$\therefore 20 = 0 + a \times 10$

$\therefore a = \frac{20}{10} = 2 \text{ m/s}^2$

∴ The acceleration of the car is  $2 \text{ m/s}^2$ .

Q.3. A ball is thrown vertically upwards with an initial velocity of 15 m/s. Find the maximum height reached by the ball.

Given:  $u = 15 \text{ m/s}$ ,  $v = 0 \text{ m/s}$

$$\boxed{C_6} = \sqrt{a^2 - (d_6)^2} = \sqrt{\left(-\frac{\sqrt{2}}{4} l\right)^2 - l^2} = \sqrt{\frac{22}{16} - 1} \cdot l = \sqrt{\frac{6}{16}} \cdot l = \boxed{\frac{\sqrt{6}}{4} l} =$$

$$= 0.61237244... l$$

Para el caso del dibujo, será:  $C_6 = 0.61237244... \times 46.9 = 28.7 \text{ mm.}$

Ángulo rectilíneo " $\alpha_3$ " del diedro formado por una cara triangular, con el plano diametral del arquimedianos, que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{\frac{1}{\tan} \alpha_3} = \frac{2 C_3}{\sqrt{4 (d_3)^2 - l^2}} = \frac{2 \times \frac{5\sqrt{6}}{12} l}{\sqrt{4 \times \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{\frac{5\sqrt{6}}{6}}{\sqrt{4 \times \frac{1}{3} - 1}} = \frac{5\sqrt{6}}{6} : \sqrt{\frac{1}{3}} =$$

$$= \frac{5}{6} \sqrt{6 : \frac{1}{3}} = \frac{5}{6} \sqrt{18} = \frac{5 \times 3 \cdot \sqrt{2}}{6} = \boxed{\frac{5\sqrt{2}}{2}} = 3.5355339$$

$$\frac{1}{\tan} \alpha_3 = 0.5484550$$

$$\boxed{\alpha_3 = 74^\circ 12' 24.6''}$$

Ángulo rectilíneo " $\alpha_6$ " del diedro formado por una cara octagonal, con el plano diametral del arquimedianos, que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [6]. (ver lám. 33).

$$\boxed{\frac{1}{\tan} \alpha_6} = \frac{2 C_6}{\sqrt{4 (d_6)^2 - l^2}} = \frac{2 \times \frac{\sqrt{6}}{4} l}{\sqrt{4 l^2 - l^2}} = \frac{\frac{\sqrt{6}}{2}}{\sqrt{3}} = \boxed{\frac{\sqrt{2}}{2}} = 0.7071068...$$

--	--	--

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation  $f(x) = \int_0^x f(t) dt$ . It is shown that  $f(x)$  is a constant function, and its value is determined by the initial condition  $f(0) = 1$ . The second part of the paper is devoted to the study of the properties of the function  $g(x)$  defined by the equation  $g(x) = \int_0^x g(t) dt$ . It is shown that  $g(x)$  is a constant function, and its value is determined by the initial condition  $g(0) = 1$ .

$$\begin{aligned}
 & \frac{1}{x} = \frac{1}{x} \\
 & \frac{1}{x} = \frac{1}{x} \\
 & \frac{1}{x} = \frac{1}{x}
 \end{aligned}$$

The third part of the paper is devoted to the study of the properties of the function  $h(x)$  defined by the equation  $h(x) = \int_0^x h(t) dt$ . It is shown that  $h(x)$  is a constant function, and its value is determined by the initial condition  $h(0) = 1$ . The fourth part of the paper is devoted to the study of the properties of the function  $k(x)$  defined by the equation  $k(x) = \int_0^x k(t) dt$ . It is shown that  $k(x)$  is a constant function, and its value is determined by the initial condition  $k(0) = 1$ .



$$\frac{1}{2} \frac{1}{2} \alpha_6 = 7,8494850$$

$$\alpha_6 = 35^\circ 15' 51,8''$$

Ángulo rectilíneo  $\varphi_{3-6}$  del diedro formado por una cara triangular y una exagonal, ambas regulares.

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{3-6}} = \alpha_3 + \alpha_6 = 74^\circ 12' 24,6'' + 35^\circ 15' 51,8'' = \boxed{109^\circ 28' 16,4''}$$

Puede obtenerse su valor directamente, por la fórmula

$$\begin{aligned} \frac{1}{2} \varphi_{3-6} &= \frac{1}{2} (\alpha_3 + \alpha_6) = \frac{\frac{1}{2} \alpha_3 + \frac{1}{2} \alpha_6}{1 - \frac{1}{2} \alpha_3 \frac{1}{2} \alpha_6} = \frac{\frac{5\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{1 - \frac{5\sqrt{2}}{2} \times \frac{\sqrt{2}}{2}} = \frac{3\sqrt{2}}{1 - \frac{10}{4}} = \\ &= \frac{3\sqrt{2}}{-\frac{6}{4}} = -\frac{3\sqrt{2}}{\frac{3}{2}} = \boxed{-2\sqrt{2}} \quad \text{y haciendo } \varphi_0 = \pi - \varphi_{3-6}, \text{ resulta} \end{aligned}$$

$$\frac{1}{2} \varphi_0 = -\frac{1}{2} \varphi_{3-6} = -(-2\sqrt{2}) = 2\sqrt{2} = 2,8284271...$$

$$\frac{1}{2} \frac{1}{2} \varphi_0 = 0,4515408$$

$$\varphi_0 = 70^\circ 31' 43,6''$$

$$\boxed{\varphi_{3-6}} = 180^\circ - 70^\circ 31' 43,6'' = \boxed{109^\circ 28' 16,4''}$$

Ángulo rectilíneo  $\varphi_{6-6}$  del diedro formado por dos caras exagonales regulares

Aplicando la fórmula general [4], (ver lám. 33), tendremos:

$$\boxed{\varphi_{6-6}} = 2\alpha_6 = 2 \times (35^\circ 15' 51,8'') = \boxed{70^\circ 31' 43,6''}$$





Puede obtenerse directamente, por la fórmula

$$\tan \varphi_{6-6} = \frac{1}{\tan 2\alpha_6} = \frac{2 \times \frac{\sqrt{2}}{2} \alpha_6}{1 - \frac{\sqrt{2}}{2}^2} = \frac{\sqrt{2}}{1 - \frac{1}{2}} = \boxed{2\sqrt{2}} =$$

$$= 2,8284271...$$

$$\tan \varphi_{6-6} = 0,4515408$$

$$\boxed{\varphi_{6-6} = 70^\circ 31' 43,6''}$$

Las fórmulas anteriores nos demuestran que los ángulos  $\varphi_{3-6}$  y  $\varphi_{6-6}$  son suplementarios, puesto que

$$\varphi_{3-6} + \varphi_{6-6} = 109^\circ 28' 16,1'' + 70^\circ 31' 43,6'' = 180^\circ = \pi$$

lo cual se deduce también de sus valores trigonométricos, ya que

$$\tan \varphi_{3-6} + \tan \varphi_{6-6} = -2\sqrt{2} + 2\sqrt{2} = 0$$

Área lateral "S" del arquimedianos

Se compone de la suma de 4 caras triangulares y 4 caras hexagonales, ambas regulares y de igual lado "l"; la superficie será pues

$$\boxed{S} = 4 \times \frac{\sqrt{3}}{4} l^2 + 4 \times \frac{6\sqrt{3}}{4} l^2 = (\sqrt{3} + 6\sqrt{3}) l^2 = \boxed{7\sqrt{3} l^2} =$$

$$= 12,12435566... l^2$$

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Volumen "V" del arquimedianos

Se compone de la suma de 4 pirámides triangulares regulares de lado "l" y altura "c<sub>3</sub>", y de otras cuatro de base exagonal regular de lado "l" y altura "c<sub>6</sub>".  
Su volumen será pues:

$$\begin{aligned}
 V &= 4 \times \frac{\sqrt{3}}{4} l^2 \times \frac{c_3}{3} + 4 \times \frac{6\sqrt{3}}{4} l^2 \times \frac{c_6}{3} = \frac{\sqrt{3}}{3} l^2 \times \frac{5\sqrt{6}}{12} l + 2\sqrt{3} l^2 \times \frac{\sqrt{6}}{4} l = \\
 &= \left( \frac{5\sqrt{18}}{36} + \frac{2\sqrt{18}}{4} \right) l^3 = \left( \frac{5\sqrt{2}}{12} + \frac{3\sqrt{2}}{2} \right) l^3 = \left( \frac{5\sqrt{2} + 18\sqrt{2}}{12} \right) l^3 = \boxed{\frac{23\sqrt{2}}{12}} l^3 \\
 &= 2,71057599 \dots l^3
 \end{aligned}$$

FIGURA CORPÓREA

Se obtiene por el acoplamiento de 4 triángulos equiláteros de lado  $l = 46,9 \text{ mm}$  y 4 exágonos regulares, también de lado "l". El acoplamiento deberá hacerse de forma que en cada vértice concurren dos exágonos y un triángulo.

NOTA.- Obsérvese que los ángulos diedros  $\varphi_{3-6}$  y  $\varphi_{6-6}$  son respectivamente iguales a los del tetraedro regular (lámin. 1) y octaedro regular (lámin. 3).





En el cuadro sinóptico que damos a continuación, están resumidos los resultados analíticos obtenidos anteriormente.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\frac{\sqrt{22}}{4} \ell$	1, 17 26 04... $\ell$
$b$	$\frac{3\sqrt{2}}{4} \ell$	1, 06 06 60... $\ell$
$c_3$	$\frac{5\sqrt{6}}{12} \ell$	1, 02 06 21... $\ell$
$c_6$	$\frac{\sqrt{6}}{4} \ell$	0, 61 23 72... $\ell$
$d_3$	$\frac{\sqrt{3}}{3} \ell$	0, 57 73 50... $\ell$
$d_6$	$1 \ell$	1, 00 00 00... $\ell$
$m$	$\frac{3\sqrt{11}}{11} \ell$	0, 90 45 34... $\ell$
$\alpha_3$	$\frac{5\sqrt{2}}{2} \ell$	$\frac{5}{2} \alpha_3 = 3, 53 55 34$ $\alpha_3 = 74^\circ 12' 24,6''$
$\alpha_6$	$\frac{\sqrt{2}}{2} \ell$	$\frac{1}{2} \alpha_6 = 0, 70 71 07$ $\alpha_6 = 35^\circ 15' 51,8''$
$\varphi_{3-6}$	$\frac{5}{2} \varphi_{3-6} = -2\sqrt{2}$	$\frac{5}{2} \varphi_{3-6} = -2, 82 84 27$ $\varphi_{3-6} = 109^\circ 28' 16,4''$
$\varphi_{6-6}$	$\frac{5}{2} \varphi_{6-6} = 2\sqrt{2}$	$\frac{5}{2} \varphi_{6-6} = 2, 82 84 27$ $\varphi_{6-6} = 70^\circ 31' 43,6''$
$S$	$7\sqrt{3} \ell^2$	12, 12 43 56... $\ell^2$
$V$	$\frac{23\sqrt{2}}{12} \ell^3$	2, 71 05 76... $\ell^3$

The following is a list of the names of the persons who have been  
 elected to the office of the Board of Directors of the

### Board of Directors

Name	Address	City
J. H. Smith	123 Main St.	New York
W. J. Brown	456 Elm St.	Boston
C. D. White	789 Oak St.	Chicago
E. F. Green	101 Pine St.	Philadelphia
G. H. Black	234 Cedar St.	San Francisco
I. K. Gold	567 Birch St.	Los Angeles
L. M. Silver	890 Spruce St.	Portland
N. O. Copper	1122 Ash St.	Seattle
P. Q. Lead	1345 Willow St.	Denver
R. S. Zinc	1567 Poplar St.	St. Louis
T. U. Iron	1789 Hickory St.	Kansas City
V. W. Steel	1901 Walnut St.	Cincinnati
X. Y. Glass	2123 Chestnut St.	Columbus

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder, en la lámina 39, a la representación gráfica del Arquimedianos VII.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, y de procesos gráficos.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_{VII} = \text{Lato del ejercicio} = 46,9 \text{ mm}$$

$$a = 1,172604 \dots \times 46,9 = 55,0 \text{ mm}$$

$$b = 1,060660 \dots \times 46,9 = 49,7 \text{ mm}$$

$$c_3 = 1,020621 \dots \times 46,9 = 47,9 \text{ mm}$$

$$c_6 = 0,612372 \dots \times 46,9 = 28,7 \text{ mm}$$

$$d_3 = 0,577350 \dots \times 46,9 = 27,1 \text{ mm}$$

$$d_6 = 1,000000 \dots \times 46,9 = 46,9 \text{ mm}$$

El orden de operaciones del trazado gráfico (láin. 39), es el siguiente:

- 1º Situar el centro O, de coordenadas O(72, 72, 85) mm.
- 2º Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio  $a = 55,0$  mm.
- 3º Representar en I, II y III la cara triangular superior 1-2-3, supuesto el poliedro colocado con dicha cara paralela a II y sin lado (2-3) perpendicular a I. (utilícese la cota " $c_3$ " en I y III).

THE [illegible] OF [illegible]

[illegible text block containing several lines of faint, mostly illegible text]

[illegible text block containing a few lines of faint, mostly illegible text]

[illegible text block containing several lines of faint, mostly illegible text]



- 4.º Representar en I, II y III la cara exagonal inferior 7 al 12, también con un lado (7-8) perpendicular a I; dicha cara es paralela a II (utilicé la cota "c<sub>6</sub>" en I y III).
- 5.º Representar en I, II y III la cara exagonal 4-3-2-6-7-8 contigua a las representadas según las operaciones 3.º, 4.º. En I la proyección es una recta (en plano es perpendicular a I). En II, se conocen los vértices 2-3-7-8; los restantes 6 y 4 están situados en una paralela a los lados 2-3, 7-8 y equidistante de éstos (la recta 6-4 es diámetro del exágono y por consiguiente, eje de simetría del mismo). Conocida la proyección de la cara en I y II, la de III es inmediata.
- 6.º Completar en I, II y III las proyecciones de las caras restantes, que no ofrecen dificultad y se deducen de las ya obtenidas anteriormente.

Como comprobación del trazado, obsérvese que los vértices 1 y 5 en I, y los 9 y 12 en III deben quedar situados en los respectivos contornos aparentes de la esfera circunscrita (radio  $a = 55 \text{ mm}$ ).

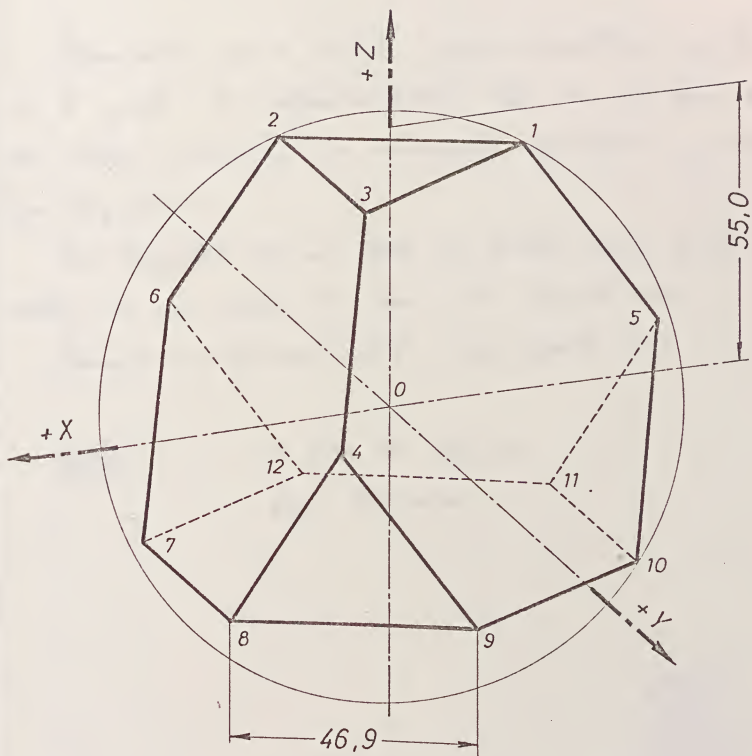
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Arquimediano VII



Fig. 1. Crystalline structure.

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano VIII, en el que en cada vértice concurren un triángulo equilátero y dos octógonos regulares.

La longitud de su lado es de 30,9 mm, y las coordenadas de su centro O, son (72, 72, 85) mm.

Dibujar en formato A3V y a escala 1:1.

DATOS

$$O (72, 72, 85) \text{ mm}$$

$$l_{VIII} = 30,9 \text{ mm}$$





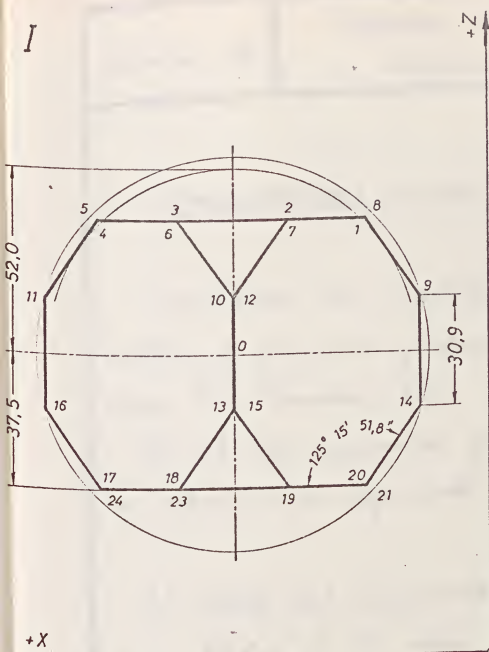
NOTES

During the summer of 1900, I spent some time in the  
vicinity of the Great Pyramids, and was fortunate enough  
to obtain a number of interesting specimens. The first of these  
was a small, dark, oval-shaped object, which I found  
in the sand near the base of the Great Pyramid. It was  
about 1/2 inch long, and 1/4 inch wide. It was very hard,  
and did not break when I tried to crush it. It was  
very smooth, and had a glossy surface. It was  
very different from anything I had ever seen before.

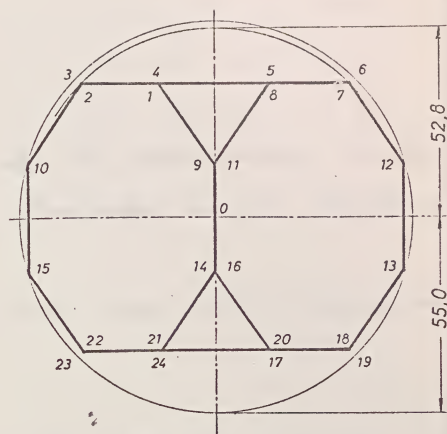
— J. H. B. —

(100)

I



III



### ARQUIMEDIANO VIII

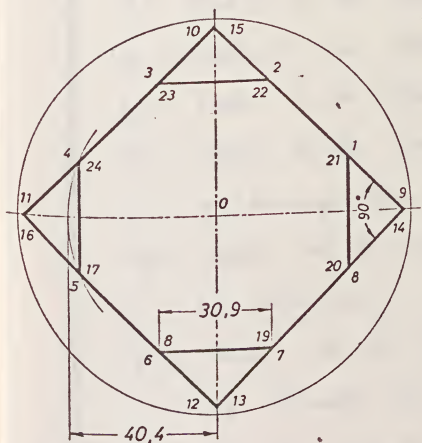
Número de caras triangulares.....  $C_3 = 8$   
 Número de caras octogonales.....  $C_8 = 6$   
 Número de vértices.....  $V = 24$   
 Número de aristas.....  $A = 36$   
 Número de caras de un ángulo sólido.....  $1P_3 + 2P_8$

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimiliano VIII, en el que en cada vértice concurren un triángulo equilátero y dos octógonos regulares.

La longitud de su lado es de 30,9 milímetros y las coordenadas de su centro O, son O (72,72,85) mm.

Dibujar en formato A3v y a escala 1:1.



	Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:						Curso
Alumno:						
Escala	Arquimiliano VIII					Lámina 40
1:1						Curso 19 -19



CONSIDERACIONES PREVIAS

Seguiremos, en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimedianos I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

$l$  = Arista del Arquimedianos VIII (dato del ejercicio).

$a$  = Radio de la esfera circunscrita

$b$  = Radio de la esfera tangente a las aristas.

$c_3$  = Radio de la esfera tangente a las caras triangulares.

$c_8$  = Radio de la esfera tangente a las caras octogonales.

$d_3$  = Radio de la circunferencia circunscrita a una cara triangular.

$d_8$  = Radio de la circunferencia circunscrita a una cara octogonal.

$m$  = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

$\alpha_3$  = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\alpha_8$  = Ángulo rectilíneo del diedro formado por una

The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1801. The letter is signed by James Madison and is addressed to the Senate and House of Representatives. The letter discusses the state of the Union and the progress of the government since the inauguration of the President.

The second part of the document is a report from the Secretary of the Treasury, dated January 1, 1801. The report is signed by Alexander Hamilton and is addressed to the Senate and House of Representatives. The report discusses the state of the Treasury and the progress of the government since the inauguration of the President.

The third part of the document is a report from the Secretary of the Navy, dated January 1, 1801. The report is signed by John Adams and is addressed to the Senate and House of Representatives. The report discusses the state of the Navy and the progress of the government since the inauguration of the President.

The fourth part of the document is a report from the Secretary of the War, dated January 1, 1801. The report is signed by Henry Knox and is addressed to the Senate and House of Representatives. The report discusses the state of the War and the progress of the government since the inauguration of the President.

The fifth part of the document is a report from the Secretary of the Interior, dated January 1, 1801. The report is signed by Thomas Mifflin and is addressed to the Senate and House of Representatives. The report discusses the state of the Interior and the progress of the government since the inauguration of the President.

The sixth part of the document is a report from the Secretary of the Agriculture, dated January 1, 1801. The report is signed by Robert H. Livingston and is addressed to the Senate and House of Representatives. The report discusses the state of the Agriculture and the progress of the government since the inauguration of the President.

The seventh part of the document is a report from the Secretary of the Commerce, dated January 1, 1801. The report is signed by John Jay and is addressed to the Senate and House of Representatives. The report discusses the state of the Commerce and the progress of the government since the inauguration of the President.

The eighth part of the document is a report from the Secretary of the Education, dated January 1, 1801. The report is signed by John Jay and is addressed to the Senate and House of Representatives. The report discusses the state of the Education and the progress of the government since the inauguration of the President.

The ninth part of the document is a report from the Secretary of the Religion, dated January 1, 1801. The report is signed by John Jay and is addressed to the Senate and House of Representatives. The report discusses the state of the Religion and the progress of the government since the inauguration of the President.

The tenth part of the document is a report from the Secretary of the Arts, dated January 1, 1801. The report is signed by John Jay and is addressed to the Senate and House of Representatives. The report discusses the state of the Arts and the progress of the government since the inauguration of the President.



cara octogonal, con el plano diametral del arquimediano que pasa por una arista de aquélla.

$\varphi_{3-8}$  = Ángulo rectilíneo del diedro formado por una cara triangular y otra octogonal.

$\varphi_{8-8}$  = Ángulo rectilíneo del diedro formado por dos caras octogonales.

$S$  = Superficie

$V$  = Volumen.

### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que se compone de 8 caras triangulares y 6 caras octogonales; 24 vértices y 36 aristas.

En cada vértice concurren un triángulo y dos octógonos, todos regulares.

Así pues, tendremos que

$$\text{ARQUIMEDIANO VIII } (1P_3 + 2P_8); C_3 = 8; C_8 = 6; V = 24; A = 36$$

Cálculo de sus magnitudes

Arista "l" del arquimediano

Dato del ejercicio

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Handwritten section header or title, centered on the page.

Second main body of handwritten text, continuing the narrative or list.

Handwritten text enclosed in a rectangular box, likely a signature or a specific note.

Handwritten text at the bottom of the page, possibly a date or a final note.

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido.

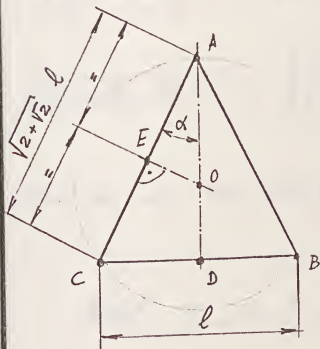


Figura 1

Dicho polígono (fig. 1) es un triángulo isósceles, cuya base  $\overline{BC}$  es la arista "l" del arquimediano (lado de la cara triangular), y sus otros dos lados iguales  $AC = AB$ , corresponden a la diagonal del octógono de lado "l" que une los extremos de dos lados consecutivos.

La geometría nos enseña que esta diagonal, es

$$\overline{AC} = \overline{AB} = \sqrt{2 + \sqrt{2}} \, l$$

De la figura se deduce:

$$\overline{AD} = \sqrt{\overline{AC}^2 - \overline{CD}^2} = \sqrt{(\sqrt{2 + \sqrt{2}} \, l)^2 - \left(\frac{l}{2}\right)^2} = \sqrt{2 + \sqrt{2} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{8 + 4\sqrt{2} - 1}{4}} \cdot l = \frac{\sqrt{7 + 4\sqrt{2}}}{2} \, l ; \text{ por lo que será:}$$

$$\cos \alpha = \frac{\overline{AD}}{\overline{AC}} = \frac{\frac{\sqrt{7 + 4\sqrt{2}}}{2} \, l}{\sqrt{2 + \sqrt{2}} \, l} = \frac{\sqrt{7 + 4\sqrt{2}}}{2\sqrt{2 + \sqrt{2}}} = \frac{1}{2} \times \sqrt{\frac{7 + 4\sqrt{2}}{2 + \sqrt{2}}} =$$

$$= \frac{1}{2} \times \sqrt{\frac{(7 + 4\sqrt{2})(2 - \sqrt{2})}{2}} = \frac{1}{2} \times \sqrt{\frac{14 + 8\sqrt{2} - 7\sqrt{2} - 8}{2}} = \frac{1}{2} \times \sqrt{\frac{6 + \sqrt{2}}{2}} = \sqrt{\frac{6 + \sqrt{2}}{8}}$$

y en consecuencia

CO

The first part of the problem is to find the area of the triangle. We can do this by using the formula for the area of a triangle, which is  $\frac{1}{2} \times \text{base} \times \text{height}$ .

In this case, the base of the triangle is 10 units and the height is 6 units. Therefore, the area of the triangle is  $\frac{1}{2} \times 10 \times 6 = 30$  square units.



The second part of the problem is to find the perimeter of the triangle.

$$P = 10 + 6 + 10 = 26$$

Therefore, the perimeter of the triangle is 26 units.

$$A = \frac{1}{2} \times 10 \times 6 = 30$$

$$P = 10 + 6 + 10 = 26$$

$$= \frac{1}{2} \times 10 \times 6 = 30$$

$$P = 10 + 6 + 10 = 26$$

$$\begin{aligned} \overline{AO} &= \boxed{m} = \frac{\overline{AE}}{\cos \alpha} = \frac{\frac{\sqrt{2+\sqrt{2}}}{2} \ell}{\frac{1}{2} \sqrt{(2+\sqrt{2}) \cdot \frac{6+\sqrt{2}}{8}}} \ell = \\ &= \frac{1}{2} \sqrt{\frac{8(2+\sqrt{2})}{6+\sqrt{2}}} \times \ell = \sqrt{\frac{2(2+\sqrt{2})}{6+\sqrt{2}}} \times \ell = \sqrt{\frac{2(2+\sqrt{2})(6-\sqrt{2})}{34}} \ell = \\ &= \sqrt{\frac{12+6\sqrt{2}-2\sqrt{2}-2}{17}} \times \ell = \boxed{\sqrt{\frac{10+4\sqrt{2}}{17}}} \times \ell = 0.95968298... \ell \end{aligned}$$

Para el caso del dibujo, será:  $m = 0.95968298... \times 30.92 = 29.7 \text{ mm.}$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33)

$$\begin{aligned} \boxed{a} &= \frac{\rho^2}{2\sqrt{\ell^2 - m^2}} = \frac{\ell^2}{2\sqrt{\ell^2 - \left(\sqrt{\frac{10+4\sqrt{2}}{17}} \ell\right)^2}} = \frac{1}{2\sqrt{1 - \frac{10+4\sqrt{2}}{17}}} \times \ell = \\ &= \frac{1}{2\sqrt{\frac{7-4\sqrt{2}}{17}}} \times \ell = \frac{1}{2} \sqrt{\frac{17}{7-4\sqrt{2}}} \times \ell = \frac{1}{2} \sqrt{\frac{17(7+4\sqrt{2})}{17}} \times \ell = \\ &= \frac{1}{2} \sqrt{7+4\sqrt{2}} \times \ell = \boxed{\frac{\sqrt{7+4\sqrt{2}}}{2}} \times \ell = 1.77882360... \ell \end{aligned}$$

Para el caso del dibujo, será:  $a = 55 \text{ mm} \quad \ell = 30.92 \text{ mm.}$

Radio "b" de la esfera tangente a las aristas

Aplicando la fórmula general [3] (ver lám. 33), tendremos:



1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and accountability in the financial process.

2. The second part outlines the procedures for handling discrepancies. If a discrepancy is identified, it should be investigated immediately. The responsible parties must provide a clear explanation and corrective action to resolve the issue. This process helps in maintaining the integrity of the financial data.

3. The third part details the requirements for the annual audit. All financial statements must be prepared in accordance with the relevant accounting standards. The audit team will conduct a thorough review of the records to ensure compliance and accuracy. Any findings will be reported to the management for their review and action.

4. The fourth part discusses the role of the internal control system. It is essential to have a robust internal control system in place to prevent errors and fraud. This includes regular monitoring and evaluation of the control measures. The system should be updated as needed to reflect changes in the business environment.

5. The fifth part covers the training and development of the financial staff. Continuous training is necessary to keep the staff updated on the latest accounting practices and regulations. The organization should provide regular training sessions and encourage staff to pursue professional certifications. This helps in building a skilled and knowledgeable financial team.

6. The sixth part addresses the communication and reporting requirements. The financial team should maintain open communication with the management and other departments. Regular reports should be submitted to provide a clear overview of the financial performance. This helps in making informed decisions and planning for the future.

7. The seventh part discusses the importance of data security. Financial data is highly sensitive and must be protected from unauthorized access. The organization should implement strong security measures, such as encryption and access controls, to safeguard the data. Regular security audits should also be conducted to ensure the effectiveness of the measures.

8. The eighth part covers the final conclusions and recommendations. The document concludes that maintaining accurate financial records is crucial for the success of the organization. The recommendations include implementing the suggested procedures and measures to improve the financial management process. The management is encouraged to take prompt action on these recommendations.

$$b = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{7} + 4\sqrt{2}}{2} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{7 + 4\sqrt{2}}{4} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{6 + 4\sqrt{2}}{4}} \times l = \frac{\sqrt{6 + 4\sqrt{2}}}{2} \times l = 1,70710678... l = \frac{(\sqrt{\frac{8}{2}} + \sqrt{\frac{4}{2}})}{2} l = \frac{2 + \sqrt{2}}{2} l$$

Para el caso del dibujo, será:  $b = 1,70710678... \times 30,92 = 52,81$

Radio "d<sub>3</sub>" de la circunferencia circunscrita a una cara triangular de lado "l"

Se demuestra en geometría, es

$$d_3 = \frac{\sqrt{3}}{3} l = 0,57735027... l$$

Para el caso del dibujo, será:  $d_3 = 0,57735027... \times 30,92 = 17,9 \text{ mm}$

Radio "d<sub>8</sub>" de la circunferencia circunscrita a una cara octogonal de lado "l"

Se demuestra en geometría, es

$$d_8 = \frac{\sqrt{2 + \sqrt{2}}}{2} l = 1,30656296... l$$

Para el caso del dibujo, será:  $d_8 = 1,30656296... \times 30,92 = 40,4 \text{ mm}$

Radio "c<sub>3</sub>" de la esfera tangente a las caras triangulares de lado "l".

Date	Particulars	Amount
1911	To Balance	100.00
1912	By Cash	50.00
1913	To Cash	25.00
1914	By Cash	75.00
1915	To Cash	100.00
1916	By Cash	150.00
1917	To Cash	200.00
1918	By Cash	250.00
1919	To Cash	300.00
1920	By Cash	350.00
1921	To Cash	400.00
1922	By Cash	450.00

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_3 &= \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\frac{\sqrt{7+4\sqrt{2}}}{2} l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{7+4\sqrt{2}}{4} - \frac{1}{3}} \times l = \\ &= \sqrt{\frac{21+12\sqrt{2}-4}{12}} \cdot l = \sqrt{\frac{17+12\sqrt{2}}{12}} \times l = \frac{\sqrt{17+12\sqrt{2}}}{2\sqrt{3}} l = \frac{\sqrt{\frac{18}{2}} + \sqrt{\frac{16}{2}}}{2\sqrt{3}} l = \\ &= \frac{3+2\sqrt{2}}{2\sqrt{3}} l = \boxed{\frac{3\sqrt{3}+2\sqrt{6}}{6}} l = 1,68\ 25\ 21\ 99\dots l \end{aligned}$$

Para el caso del dibujo, será:  $C_3 = 1,68\ 25\ 21\ 99\dots \times 30,92 = 52,0\ \text{mm}$

Radio "C<sub>8</sub>" de la esfera tangente a las caras octogonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33), tendremos:

$$\begin{aligned} C_8 &= \sqrt{a^2 - (d_8)^2} = \sqrt{\left(\frac{\sqrt{7+4\sqrt{2}}}{2} l\right)^2 - \left(\frac{\sqrt{2+\sqrt{2}}}{2} l\right)^2} = \sqrt{\frac{7+4\sqrt{2}}{4} - \frac{2+\sqrt{2}}{2}} \times l = \\ &= \sqrt{\frac{7+4\sqrt{2}-4-2\sqrt{2}}{4}} \cdot l = \sqrt{\frac{3+2\sqrt{2}}{4}} \cdot l = \frac{\sqrt{3+2\sqrt{2}}}{2} l = \frac{\sqrt{\frac{4}{2}} + \sqrt{\frac{2}{2}}}{2} l = \\ &= \boxed{\frac{1+\sqrt{2}}{2}} l = 1,21\ 21\ 06\ 78\dots l \end{aligned}$$

Para el caso del dibujo, será:  $C_8 = 1,21\ 21\ 06\ 78\dots \times 30,92 = 37,5\ \text{mm}$

Ángulo rectilíneo "α<sub>3</sub>" del diedro formado por una cara triangular, con el plano diametral del arquimedianos que pasa por una arista de aquella.





Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{\frac{1}{t_2} \alpha_2} = \frac{2 c_3}{\sqrt{4 (d_3)^2 - l^2}} = \frac{2 \times \frac{3\sqrt{3} + 2\sqrt{6}}{6} l}{\sqrt{4 \left(\frac{\sqrt{3}}{3} l\right)^2 - l^2}} = \frac{3\sqrt{3} + 2\sqrt{6}}{3\sqrt{4 \times \frac{1}{3} - 1}} = \frac{3\sqrt{3} + 2\sqrt{6}}{3\sqrt{\frac{1}{3}}} =$$

$$= \frac{(3\sqrt{3} + 2\sqrt{6}) \times \sqrt{3}}{3} = \frac{9 + 2\sqrt{18}}{3} = \frac{9 + 6\sqrt{2}}{3} = \boxed{3 + 2\sqrt{2}} = 5,82842712...$$

$$\frac{1}{t_2} \alpha_2 = 0,7655513$$

$$\alpha_2 = 80^\circ 15' 51,8''$$

Ángulo rectilíneo " $\alpha_8$ " del diedro formado por una cara octogonal, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33).

$$\boxed{\frac{1}{t_8} \alpha_8} = \frac{2 c_8}{\sqrt{4 (d_8)^2 - l^2}} = \frac{2 \times \frac{1+\sqrt{2}}{2} l}{\sqrt{4 \left(\frac{\sqrt{2}+\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{1+\sqrt{2}}{\sqrt{4 \times \frac{2+\sqrt{2}}{2} - 1}} =$$

$$= \frac{1+\sqrt{2}}{\sqrt{4+2\sqrt{2}-1}} = \frac{1+\sqrt{2}}{\sqrt{3+2\sqrt{2}}} = \frac{1+\sqrt{2}}{\left(\sqrt{\frac{4}{2}} + \sqrt{\frac{2}{2}}\right)} = \frac{1+\sqrt{2}}{(\sqrt{2}+1)} =$$

$$= \frac{(\sqrt{2}+1)(\sqrt{2}-1)}{1} = \frac{1}{1} = 1$$

$$\alpha_8 = 45^\circ$$

<p>             No. <span style="border: 1px solid black; padding: 2px;">100</span> </p>	<p>             Date <span style="border: 1px solid black; padding: 2px;">10/10/1910</span> </p> <p>             To <span style="border: 1px solid black; padding: 2px;">The Hon. Secy. of the Navy</span> </p> <p>             For <span style="border: 1px solid black; padding: 2px;">\$100.00</span> </p> <p>             By <span style="border: 1px solid black; padding: 2px;">J. M. Smith</span> </p> <p>             Total <span style="border: 1px solid black; padding: 2px;">\$100.00</span> </p>	<p>             Received of <span style="border: 1px solid black; padding: 2px;">J. M. Smith</span> </p> <p>             the sum of <span style="border: 1px solid black; padding: 2px;">\$100.00</span> </p> <p>             for <span style="border: 1px solid black; padding: 2px;">\$100.00</span> </p> <p>             Total <span style="border: 1px solid black; padding: 2px;">\$100.00</span> </p>
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Angulo rectilíneo " $\varphi_{3-8}$ " del diedro formado por una cara triangular y otra octogonal, ambas regulares.

Aplicando la fórmula general [4] (ver lám. 33), tendremos:

$$\boxed{\varphi_{3-8}} = \alpha_3 + \alpha_8 = 80^\circ 15' 51,8'' + 45^\circ = \boxed{125^\circ 15' 51,8''}$$

Puede obtenerse directamente, de la siguiente manera:

$$\begin{aligned} \boxed{\frac{1}{2} \varphi_{3-8}} &= \frac{1}{2} (\alpha_3 + \alpha_8) = \frac{\frac{1}{2} \alpha_3 + \frac{1}{2} \alpha_8}{1 - \frac{1}{2} \alpha_3 - \frac{1}{2} \alpha_8} = \frac{3 + 2\sqrt{2} + 1}{1 - (3 + 2\sqrt{2}) \times 1} = \frac{4 + 2\sqrt{2}}{1 - 3 - 2\sqrt{2}} = \\ &= \frac{4 + 2\sqrt{2}}{-2 - 2\sqrt{2}} = - \frac{2 + \sqrt{2}}{\sqrt{2} + 1} = - \frac{(2 + \sqrt{2})(\sqrt{2} - 1)}{1} = - (2\sqrt{2} + 2 - 2 - \sqrt{2}) = \boxed{-\sqrt{2}} \end{aligned}$$

y haciendo  $\alpha_0 = \pi - \varphi_{3-8}$ , será:  $\frac{1}{2} \alpha_0 = -\frac{1}{2} \varphi_{3-8} = -(-\sqrt{2}) = \sqrt{2}$

$$\frac{1}{2} \frac{1}{2} \alpha_0 = \frac{1}{2} \frac{1}{2} 3 = 0.15 \ 05 \ 15 \ 0$$

$$\alpha_0 = 54^\circ 44' 8,2''$$

$$\boxed{\varphi_{3-8}} = 180^\circ - 54^\circ 44' 8,2'' = \boxed{125^\circ 15' 51,8''}$$

Angulo rectilíneo " $\varphi_{8-8}$ " del diedro formado por dos caras octogonales regulares

Aplicando la fórmula general [4] (ver lám. 33), tendremos

$$\boxed{\varphi_{8-8}} = 2 \alpha_8 = 2 \times 45^\circ = \boxed{90^\circ}$$

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Área lateral "S" del arquimedianos

Se compone de la suma de 8 caras triangulares y 6 caras octogonales, ambas regulares y de igual lado "l".

Siendo "d<sub>8</sub>" el radio de la circunferencia circunscrita al octógono de lado "l", su apotema será:

$$\begin{aligned} \text{apotema} &= \sqrt{(d_8)^2 - \left(\frac{l}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{2+\sqrt{2}}{2}} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{2+\sqrt{2}}{2} - \frac{1}{4}} \cdot l = \\ &= \sqrt{\frac{4+2\sqrt{2}-1}{4}} \cdot l = \frac{\sqrt{3+2\sqrt{2}}}{2} \cdot l, \quad \text{y el área pedida será:} \end{aligned}$$

$$[S] = 8 \times \frac{\sqrt{3}}{4} l^2 + 6 \times \frac{8l}{2} \times \frac{\sqrt{3+2\sqrt{2}}}{2} \cdot l = (2\sqrt{3} + 12\sqrt{3+2\sqrt{2}}) \times l^2 =$$

$$= [2\sqrt{3} + 12\left(\sqrt{\frac{4}{2}} + \sqrt{\frac{2}{2}}\right)] \cdot l^2 = (2\sqrt{3} + 12\sqrt{2} + 12) \cdot l^2 = \boxed{2 \cdot (\sqrt{3} + 6\sqrt{2} + 6) l^2} =$$

$$= 32,43 \ 46 \ 64 \ 34 \dots l^2$$

Volumen "V" del arquimedianos.

Se compone de la suma de 8 pirámides triangulares regulares de lado "l" y altura "c<sub>3</sub>", y de 6 pirámides octogonales regulares de lado "l" y altura "c<sub>8</sub>". Su volumen será pues:

$$\begin{aligned} [V] &= 2\sqrt{3} l^2 \times \frac{c_3}{3} + 12(\sqrt{2}+1) l^2 \times \frac{c_8}{3} = 2\sqrt{3} l^2 \times \frac{3\sqrt{3} + 2\sqrt{6}}{6 \times 3} \cdot l + \\ &+ 12(\sqrt{2}+1) l^2 \times \frac{\sqrt{2}+1}{2 \times 3} l = \left(\frac{9+6\sqrt{2}}{9} + 2(\sqrt{2}+1)^2\right) \times l^3 = \left(\frac{3+2\sqrt{2}}{3} + \right. \end{aligned}$$





$$+ \frac{6(\sqrt{2}+1)^2}{3} l^3 = \frac{3+2\sqrt{2}+6(\sqrt{2}+1)^2}{3} l^3 = \frac{3+2\sqrt{2}+6(2+1+2\sqrt{2})}{3} l^3 =$$

$$= \frac{3+2\sqrt{2}+18+12\sqrt{2}}{3} l^3 = \frac{21+14\sqrt{2}}{3} l^3 = \boxed{\frac{7(3+2\sqrt{2})}{3}} l^3 = 13,59966328..l^3$$

FIGURA CORPÓREA

Se obtiene por el acoplamiento de 8 triángulos equiláteros de lado  $l = 30,9 \text{ mm}$  y 6 octógonos regulares, también de lado " $l$ ". El acoplamiento deberá hacerse de forma que en cada vértice concueran 2 octógonos y un triángulo.

En el cuadro sinóptico que damos a continuación, están resumidos los resultados analíticos obtenidos anteriormente.

No.	Name	Age
1	John Smith	25
2	Mary Jones	30
3	James Brown	28
4	Elizabeth White	22
5	Robert Black	35
6	Sarah Green	27
7	William Hall	32
8	Anna King	24
9	George Lee	31
10	Margaret Taylor	29
11	Thomas Young	26
12	Elizabeth Clark	33
13	Richard Adams	21

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\frac{\sqrt{7} + 4\sqrt{2}}{2} l$	1. 77 88 24.... $l$
$b$	$\frac{2 + \sqrt{2}}{2} l$	1. 70 71 07.... $l$
$c_3$	$\frac{3\sqrt{3} + 2\sqrt{6}}{6} l$	1. 68 25 92.... $l$
$c_8$	$\frac{\sqrt{2} + 1}{2} l$	1. 21 21 07.... $l$
$d_3$	$\frac{\sqrt{3}}{3} l$	0. 57 73 50.... $l$
$d_8$	$\sqrt{\frac{2 + \sqrt{2}}{2}} l$	1. 30 65 63.... $l$
$m$	$\sqrt{\frac{10 + 4\sqrt{2}}{17}} l$	0. 95 96 83.... $l$
$\alpha_3$	$\frac{1}{2} \alpha_3 = 3 + 2\sqrt{2}$	$\frac{1}{2} \alpha_3 = 5.828437...$ $\alpha_3 = 80^\circ 15' 51.8''$
$\alpha_8$	$\frac{1}{2} \alpha_8 = 1$	$\alpha_8 = 45^\circ$
$\varphi_{3-8}$	$\frac{1}{2} \varphi_{3-8} = -\sqrt{2}$	$\frac{1}{2} \varphi_{3-8} = -1.414214...$ $\varphi_{3-8} = 125^\circ 15' 51.8''$
$\varphi_{8-8}$	$\frac{1}{2} \varphi_{8-8} = \infty$	$\varphi_{8-8} = 90^\circ$
$S$	$2(\sqrt{3} + 6\sqrt{2} + 6) l^2$	32. 43 46 64.... $l^2$
$V$	$\frac{7(3 + 2\sqrt{2})}{3} l^3$	13. 59 96 63.... $l^3$





PROCESO GRÁFICO - ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder, en la lámina 40, a la representación gráfica del Arquimediano VIII.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, y de procesos gráficos.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_{VIII} = \text{Dato del ejercicio} = 30,9 \text{ mm}$$

$$a = 1,748824... \times 30,92 = 55,0 \text{ mm}$$

$$b = 1,707107... \times 30,92 = 52,8 \text{ mm}$$

$$c_3 = 1,682592... \times 30,92 = 52,0 \text{ mm}$$

$$c_8 = 1,212107... \times 30,92 = 37,5 \text{ mm}$$

$$d_3 = 0,577350... \times 30,92 = 17,9 \text{ mm}$$

$$d_8 = 1,306563... \times 30,92 = 40,4 \text{ mm}$$

El orden de operaciones del trazado gráfico (lámin. 40), es el siguiente:

1.º Situar el centro O, de coordenadas O (72, 72, 85) mm.

2.º Dibujar en I, II y III, las proyecciones de la esfera circunscrita, de radio  $a = 55,0$  mm.

3.º Representar en I, II y III las caras octogonales superiores 1 al 8 y la inferior 17 al 24, supuesto el poliedro colocado con dichas caras paralelas a II, y un

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lado perpendicular a I (utilizase la cota " $c_8$ " en I y III, y la  $d_8$  en II).

1.º Obtener en I, II y III, las proyecciones de las aristas 9-14, 10-15, 11-16 y 12-13, -perpendiculares a II, que en esta proyección se reducen a un punto cada una; estos puntos son vértices de un cuadrado, prolongación de los lados 3-4, 5-6, 7-8, 1-2 (en II). En I y III dichas aristas son perpendiculares a los ejes  $x$  e  $y$  respectivamente y sus extremos equidistan del plano diametral paralelo a II.

Con las anteriores construcciones puede completarse las representaciones en I, II y III del quericimedianos arquimedianos, con una ordenada unión de los vértices obtenidos.

Como comprobación del trazado, obsérvese que los vértices 9, 11, 14 y 16 en II, y los 10, 12, 13, 15 en III, deben quedar situados en los respectivos contornos aparentes de la esfera circunscrita (radio  $a = 55,0$  mm).



[The following text is extremely faint and largely illegible. It appears to be a formal document or report, possibly containing a title, a date, and several paragraphs of text. The text is written in a cursive or semi-cursive hand.]

[Illegible text block 1]

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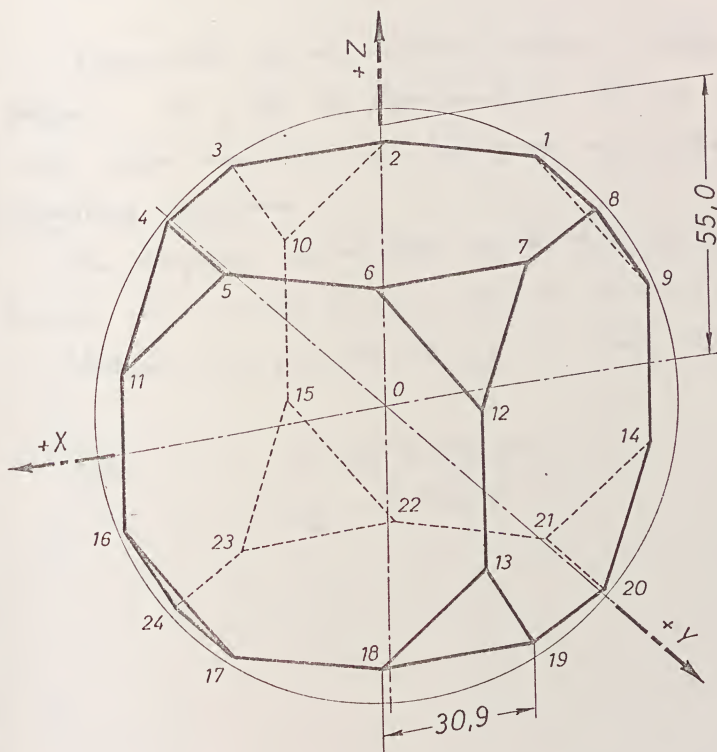
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Arquimediano VIII





Diagram of the Earth

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimédiano IX, en el que en cada vértice concurren un triángulo equilátero y dos decágonos regulares.

La longitud de su lado es de 18,5 mm, y las coordenadas de su centro O, son (72, 72, 85) mm.

... Dibujar en formato A3V y a escala 1:1.

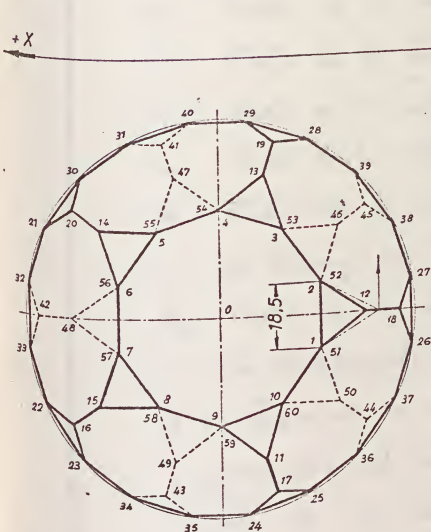
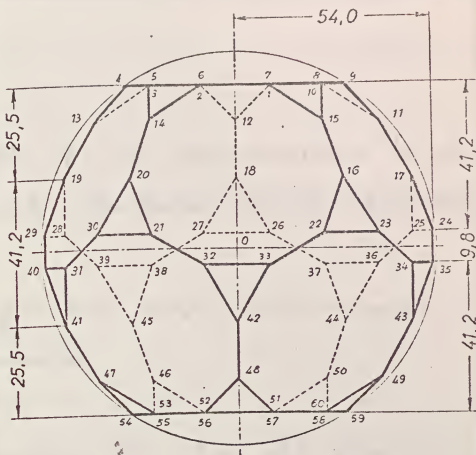
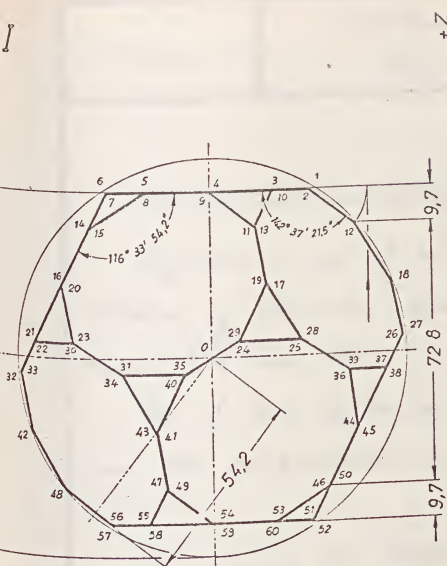
DATOS

$$O (72, 72, 85) \text{ mm}$$

$$l_{IX} = 18,5 \text{ mm}$$

$$l = 18,53$$





### ARQUIMEDIANO IX

Número de caras triangulares.....	$C_3 = 20$
Número de caras decagonales.....	$C_{10} = 12$
Número de vértices.....	$V = 60$
Número de aristas.....	$A = 90$
Número de caras de un ángulo sólido.....	$1 P_3 + 2 P_6$

### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedian IX, en el que en cada vértice concurren un triángulo equilátero y dos decágonos regulares.

La longitud de su lado es de 18,5 milímetros y las coordenadas de su centro O, son O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela
Fecha:					Curso
Alumno:					
Escala					

Arquimedian IX

Lámina 41

Curso 13 - 19



1. The first diagram is a circle with internal lines forming a hexagram-like pattern. The lines connect points on the circumference to points inside the circle, creating a series of triangles and other geometric shapes.

2. The second diagram is a circle with internal lines forming a hexagram-like pattern. The lines connect points on the circumference to points inside the circle, creating a series of triangles and other geometric shapes.



21	22	23	24
25	26	27	28



CONSIDERACIONES PREVIAS

Seguiremos, en el estudio de este arquimediano, las directrices y fórmulas generales planteadas en el "Arquimediano I" (lámin. 33).

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

- $l$  = Arista del Arquimediano IX (dato del ejercicio).
- $a$  = Radio de la esfera circunscrita.
- $b$  = Radio de la esfera tangente a las aristas.
- $c_3$  = Radio de la esfera tangente a las caras triangulares.
- $c_{10}$  = Radio de la esfera tangente a las caras decagonales.
- $d_3$  = Radio de la circunferencia circunscrita a una cara triangular.
- $d_{10}$  = Radio de la circunferencia circunscrita a una cara decagonal.
- $m$  = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.
- $\alpha_3$  = Ángulo rectilíneo del diedro formado por una cara triangular, con el plano diametral del arquimediano que pasa por una arista de aquella.
- $\alpha_{10}$  = Ángulo rectilíneo del diedro formado por una cara decagonal, con el plano diametral del ar-



quimediano que pasa por una arista de aquella.

$\varphi_{3-10}$  = Angulo rectilíneo del diedro formado por una cara triangular y otra decagonal.

$\varphi_{10-10}$  = Angulo rectilíneo del diedro formado por dos caras decagonales.

S = Superficie

V = Volumen.

### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que se compone de 20 caras triangulares y 12 caras decagonales; 60 vértices y 90 aristas.

En cada vértice concurren un triángulo y dos decágonos, todos regulares.

Así pues, tendremos que

$$\text{ARQUIMEDIANO IX } (1 P_3 + 2 P_{10}); C_3 = 20; C_{10} = 12; V = 60; A = 90$$

### Cálculo de sus magnitudes

Arista "l" del arquimedianos

Dato del ejercicio

No.

NAME

DATE

I hereby certify that the above named person is a member of the  
of the County of \_\_\_\_\_ State of \_\_\_\_\_  
and is entitled to the same rights and privileges as other members of the same.  
Witness my hand and seal this \_\_\_\_\_ day of \_\_\_\_\_ 19\_\_\_\_.

\_\_\_\_\_  
Secretary

I hereby certify that the above named person is a member of the  
of the County of \_\_\_\_\_ State of \_\_\_\_\_  
and is entitled to the same rights and privileges as other members of the same.  
Witness my hand and seal this \_\_\_\_\_ day of \_\_\_\_\_ 19\_\_\_\_.

\_\_\_\_\_  
Secretary

\_\_\_\_\_  
Secretary

\_\_\_\_\_  
Secretary

\_\_\_\_\_  
Secretary

\_\_\_\_\_  
Secretary

\_\_\_\_\_  
Secretary

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido.

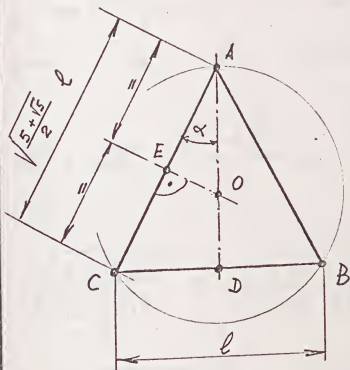


Figura 1

Dicho polígono (Fig. 1), es un triángulo isósceles, cuya base  $\overline{BC}$  es la arista "l" del arquimedianos (lado de la cara triangular), y sus otros dos lados iguales  $\overline{AC} = \overline{AB}$ , corresponden a la diagonal del decágono de lado "l" que une los extremos de dos lados consecutivos.

Se demuestra en Geometría que el lado " $l_{10}$ " de un decágono regular, inscrito en una circunferencia de radio "R," tiene el valor de

$$l_{10} = \frac{\sqrt{5} - 1}{2} R \quad [1]$$

Igualmente, el lado " $l_5$ " de un pentágono regular, en función de "R" es a su vez

$$l_5 = \frac{\sqrt{10 - 2\sqrt{5}}}{2} R \quad [2]$$

Como la diagonal de un decágono regular que une los extremos de dos lados consecutivos del mismo, es un pentágono regular, ambos inscritos en una



Let us consider the following problem: A body of mass  $m$  moves with a constant velocity  $v$  in a straight line. The force acting on it is zero. Find the work done by the force.

Solution: The work done by a force  $F$  in moving a body through a distance  $s$  is given by the formula  $W = F \cdot s \cdot \cos \theta$ , where  $\theta$  is the angle between the force and the displacement. In this case, the force is zero, so the work done is also zero.



Another example: A body of mass  $m$  moves in a circular path of radius  $r$  with a constant angular velocity  $\omega$ . Find the work done by the centripetal force.

Solution: The centripetal force acts towards the center of the circle, perpendicular to the displacement. Therefore, the work done by the centripetal force is zero.

Let us now consider a more complex problem: A body of mass  $m$  moves in a circular path of radius  $r$  with a constant speed  $v$ . Find the work done by the centripetal force over one complete revolution.

misma circunferencia, tendremos que [2]

$$\overline{AC} = l_5 = \frac{\sqrt{10-2\sqrt{5}}}{2} R$$

y sustituyendo "R" por su valor obtenido de [1], en la que  $l_{10} = l$ , tendremos

$$\begin{aligned}\overline{AC} = \overline{AB} &= \frac{\sqrt{10-2\sqrt{5}}}{2} R = \frac{\sqrt{10-2\sqrt{5}}}{2} \times \frac{2}{\sqrt{5}-1} \times l = \frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1} l = \\ &= \sqrt{\frac{2(5-\sqrt{5})}{(\sqrt{5}-1)^2}} l = \sqrt{\frac{2(5-\sqrt{5})}{5+1-2\sqrt{5}}} l = \sqrt{\frac{2(5-\sqrt{5})}{6-2\sqrt{5}}} l = \sqrt{\frac{5-\sqrt{5}}{3-\sqrt{5}}} l = \\ &= \sqrt{\frac{(5-\sqrt{5})(3+\sqrt{5})}{4}} l = \sqrt{\frac{15-3\sqrt{5}+5\sqrt{5}-5}{4}} l = \sqrt{\frac{10+2\sqrt{5}}{4}} l = \sqrt{\frac{5+\sqrt{5}}{2}} l\end{aligned}$$

De la figura se deduce:

$$\begin{aligned}\overline{AD} &= \sqrt{\overline{AC}^2 - \overline{CD}^2} = \sqrt{\left(\sqrt{\frac{5+\sqrt{5}}{2}} l\right)^2 - \left(\frac{l}{2}\right)^2} = \sqrt{\frac{5+\sqrt{5}}{2} - \frac{1}{4}} l = \\ &= \sqrt{\frac{10+2\sqrt{5}-1}{4}} l = \sqrt{\frac{9+2\sqrt{5}}{4}} l = \frac{\sqrt{9+2\sqrt{5}}}{2} l \text{ por lo que será:}\end{aligned}$$

$$\cos \alpha = \frac{\overline{AD}}{\overline{AC}} = \frac{\sqrt{9+2\sqrt{5}}}{2} l : \sqrt{\frac{5+\sqrt{5}}{2}} l = \frac{1}{2} \sqrt{(9+2\sqrt{5}) : \frac{5+\sqrt{5}}{2}} =$$

$$= \frac{1}{2} \sqrt{\frac{2(9+2\sqrt{5})}{5+\sqrt{5}}} = \frac{1}{2} \sqrt{\frac{2(9+2\sqrt{5})(5-\sqrt{5})}{20}} = \frac{1}{2} \sqrt{\frac{45+10\sqrt{5}-9\sqrt{5}-10}{10}} =$$

$$= \frac{1}{2} \sqrt{\frac{35+\sqrt{5}}{10}} = \sqrt{\frac{35+\sqrt{5}}{40}} \quad \text{y en consecuencia}$$

--	--	--


$$\begin{aligned}\overline{AO} &= \boxed{m} = \frac{\overline{AE}}{\cos \alpha} = \frac{\overline{AC}}{2 \cos \alpha} = \frac{\sqrt{\frac{5+\sqrt{5}}{2}} l}{2} : 2 \sqrt{\frac{35+\sqrt{5}}{40}} = \\&= \sqrt{\frac{5+\sqrt{5}}{2}} : \frac{35+\sqrt{5}}{10} \times l = \sqrt{\frac{(5+\sqrt{5}) \times 10}{(35+\sqrt{5}) \times 2}} \times l = \sqrt{\frac{5(5+\sqrt{5})(35-\sqrt{5})}{35^2-5}} \times l = \\&= \sqrt{\frac{5 \times 35 + 35\sqrt{5} - 5\sqrt{5} - 5}{35 \times 7 - 1}} \times l = \sqrt{\frac{190 + 30\sqrt{5}}{244}} \times l = \sqrt{\frac{85 + 15\sqrt{5}}{122}} \times l = \\&= \boxed{\sqrt{\frac{5(17+3\sqrt{5})}{122}}} \times l = 0,98\ 57\ 21\ 90\dots l\end{aligned}$$

Para el caso del dibujo, será:  $m = 0,98\ 57\ 21\ 9\dots \times 18,53 = 18,3\ \text{m m.}$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33)

$$\begin{aligned}\boxed{a} &= \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\sqrt{\frac{5(17+3\sqrt{5})}{122}} l\right)^2}} = \frac{1}{2\sqrt{1 - \frac{5(17+3\sqrt{5})}{122}}} \times l = \\&= \frac{1}{2\sqrt{\frac{122 - 85 - 15\sqrt{5}}{122}}} \times l = \frac{1}{2\sqrt{\frac{37 - 15\sqrt{5}}{122}}} \times l = \frac{1}{2} \sqrt{\frac{122}{37 - 15\sqrt{5}}} \times l = \\&= \frac{1}{2} \sqrt{\frac{122 \times (37 + 15\sqrt{5})}{37^2 - 15^2 \times 5}} \times l = \sqrt{\frac{122(37 + 15\sqrt{5})}{244}} \times l = \frac{1}{2} \sqrt{\frac{37 + 15\sqrt{5}}{2}} \times l = \\&= \boxed{\sqrt{\frac{37 + 15\sqrt{5}}{8}}} \times l = 2,96\ 94\ 49\ 01\dots l\end{aligned}$$

Para el caso del dibujo, será:  $a = 55,0\ \text{m m}$   $l = 18,52\ 19\ 5 \approx 18,53\ \text{m m}$

Date	Particulars	Amount
	By Balance b/d	
	To Cash	
	To Bank	
	To Sales	
	To Income	
	To Profit	
	To Dividend	
	To Interest	
	To Rent	
	To Commission	
	To Brokerage	
	To Freight	
	To Insurance	
	To Taxes	
	To Salaries	
	To Wages	
	To Expenses	
	To Loss	
	To Drawings	
	To Balance c/d	



Radio "b" de la esfera tangente a las aristas

Se obtiene aplicando la fórmula general [3] (ver lám. 33).

$$\begin{aligned}
 b &= \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\sqrt{\frac{37+15\sqrt{5}}{8}} \cdot l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{1}{4}} \cdot l = \\
 &= \sqrt{\frac{37+15\sqrt{5}-2}{8}} \cdot l = \sqrt{\frac{35+15\sqrt{5}}{8}} \cdot l = \sqrt{\frac{5 \cdot (7+3\sqrt{5})}{8}} \cdot l = \sqrt{\frac{5}{8}} \cdot \sqrt{7+3\sqrt{5}} \cdot l = \\
 &= \sqrt{\frac{5}{8}} \cdot \left(\sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}}\right) \cdot l = \left(\sqrt{\frac{45}{16}} + \sqrt{\frac{25}{16}}\right) \cdot l = \left(\frac{3\sqrt{5}}{4} + \frac{5}{4}\right) l = \frac{3\sqrt{5}+5}{4} \cdot l = \\
 &= 2,92705098... l
 \end{aligned}$$

Para el caso del dibujo, será:  $b = 2,92705298... \times 18,52 = 54,2 \text{ mm}$

Radio "d<sub>3</sub>" de la circunferencia circunscrita a una cara triangular

Se demuestra en Geometría, es:

$$d_3 = \frac{\sqrt{3}}{3} l = 0,57735027... l$$

Para el caso del dibujo, será:  $d_3 = 0,57735029... \times 18,53 = 10,7 \text{ mm}$

Radio "d<sub>10</sub>" de la circunferencia circunscrita a una cara dicagonal

Se demuestra en Geometría, es:

EO



$$d_{10} = \frac{\sqrt{5} + 1}{2} l = 1.61803399... l$$

Para el caso del dibujo, será:  $d_{10} = 1.61803399... \times 18.53 = 30.0 \text{ mm}$ .

Radio "c<sub>3</sub>" de la esfera tangente a las caras triangulares de lado "l".

Aplicando la fórmula general [2] (ver lám. 33),

$$\begin{aligned} c_3 &= \sqrt{a^2 - (d_3)^2} = \sqrt{\left(\sqrt{\frac{37+15\sqrt{5}}{8}} \times l\right)^2 - \left(\frac{\sqrt{3}}{3} l\right)^2} = \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{1}{3}} \times l = \\ &= \sqrt{\frac{111+45\sqrt{5}-8}{24}} \times l = \sqrt{\frac{103+45\sqrt{5}}{24}} \times l = \frac{\sqrt{103+45\sqrt{5}}}{\sqrt{24}} \times l = \frac{\sqrt{\frac{125}{2}} + \sqrt{\frac{81}{2}}}{\sqrt{24}} l = \\ &= \left(\sqrt{\frac{125}{48}} + \sqrt{\frac{81}{48}}\right) l = \left(\frac{5\sqrt{5}}{4\sqrt{3}} + \frac{9}{4\sqrt{3}}\right) l = \left(\frac{5\sqrt{5}+9}{4\sqrt{3}}\right) l = \boxed{\frac{5\sqrt{5}+9\sqrt{3}}{12}} l = \end{aligned}$$

Para el caso del dibujo, será:

$$c_3 = 2.91278117... \times 18.53 = 54.0 \text{ mm}$$

Radio "c<sub>10</sub>" de la esfera tangente a las caras decagonales de lado "l".

Aplicando la fórmula general [2] (ver lám. 33),

$$\begin{aligned} c_{10} &= \sqrt{a^2 - (d_{10})^2} = \sqrt{\left(\sqrt{\frac{37+15\sqrt{5}}{8}} \times l\right)^2 - \left(\frac{\sqrt{5}+1}{2} l\right)^2} = \\ &= \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{5+1+2\sqrt{5}}{4}} \times l = \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{6+2\sqrt{5}}{4}} \times l = \end{aligned}$$

10

10

10

$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$$

10

10

10

10

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10

10

10

10

$$= \sqrt{\frac{37 + 15\sqrt{5} - 12 - 4\sqrt{5}}{8}} \times \ell = \sqrt{\frac{25 + 11\sqrt{5}}{8}} \times \ell = 2,48\ 98\ 98\ 3\dots \ell$$

Para el caso del dibujo, será:  $C_{10} = 2,48\ 98\ 98\ 3\dots \times 18,53 = 46,1\text{ mm}$

Ángulo rectilíneo " $\alpha_3$ " del diedro formado por una cara triangular, con el plano diametral del arquimédiano que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{tg\ \alpha_3} = \frac{2\ c_3}{\sqrt{4\ (d_3)^2 - \ell^2}} = \frac{2 \times \frac{5\sqrt{15} + 9\sqrt{3}}{12} \ell}{\sqrt{4\ \left(\frac{\sqrt{3}}{3} \ell\right)^2 - \ell^2}} = \frac{5\sqrt{15} + 9\sqrt{3}}{6\sqrt{4 \times \frac{1}{3} - 1}} =$$

$$= \frac{5\sqrt{15} + 9\sqrt{3}}{6\sqrt{\frac{1}{3}}} = \frac{(5\sqrt{15} + 9\sqrt{3}) \times \sqrt{3}}{6} = \frac{5\sqrt{45} + 9 \times 3}{6} = \frac{15\sqrt{5} + 27}{6} =$$

$$= \frac{5\sqrt{5} + 9}{2} = 10,09\ 01\ 69\ 95\dots$$

$$tg\ tg\ \alpha_3 = 1,00\ 38\ 98\ 5$$

$$\boxed{\alpha_3 = 84^\circ\ 20'\ 34,4''}$$

Ángulo rectilíneo " $\alpha_{10}$ " del diedro formado por una cara decagonal, con el plano diametral del arquimédiano que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33).



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1. The first part of the paper is devoted to a study of the

properties of the function  $f(x)$  defined by the equation

$f(x) = \int_0^x f(t) dt$  for  $x \in [0, 1]$ . It is shown that the function

is continuous and differentiable on the interval  $[0, 1]$ .

2. In the second part, we consider the problem of finding the

extremum of the functional

under the constraint

where  $\lambda$  is a Lagrange multiplier.

3. The third part of the paper is devoted to a study of the

properties of the function  $f(x)$  defined by the equation

for  $x \in [0, 1]$ . It is shown that the function

$$\boxed{t_f \alpha_{10}} = \frac{2 C_0}{\sqrt{4 (d_{10})^2 - l^2}} = \frac{2 \sqrt{\frac{25+11\sqrt{5}}{8}} \times l}{\sqrt{4 \left(\frac{\sqrt{5}+1}{2} l\right)^2 - l^2}} = \frac{\sqrt{\frac{25+11\sqrt{5}}{2}}}{\sqrt{4 \times \frac{(\sqrt{5}+1)^2}{4} - 1}} =$$

$$= \frac{\sqrt{\frac{25+11\sqrt{5}}{2}}}{\sqrt{(\sqrt{5}+1)^2 - 1}} = \frac{\sqrt{\frac{25+11\sqrt{5}}{2}}}{\sqrt{5+1+2\sqrt{5}-1}} = \frac{\sqrt{\frac{25+11\sqrt{5}}{2}}}{\sqrt{5+2\sqrt{5}}} = \sqrt{\frac{25+11\sqrt{5}}{2(5+2\sqrt{5})}} = \sqrt{\frac{(25+11\sqrt{5})(5-2\sqrt{5})}{2(25-20)}} =$$

$$= \sqrt{\frac{125 + 55\sqrt{5} - 50\sqrt{5} - 110}{10}} = \sqrt{\frac{15 + 5\sqrt{5}}{10}} = \sqrt{\frac{3+\sqrt{5}}{2}} = \frac{\sqrt{3+\sqrt{5}}}{\sqrt{2}} = \frac{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}}{\sqrt{2}} =$$

$$= \sqrt{\frac{5}{4}} + \sqrt{\frac{1}{4}} = \frac{\sqrt{5}}{2} + \frac{1}{2} = \boxed{\frac{\sqrt{5}+1}{2}} = 1,61803399...$$

$$t_f t_f \alpha_{10} = 0,2089876$$

$$\alpha_{10} = 58^\circ 16' 57,1''$$

Ángulo rectilíneo " $\varphi_{3-10}$ " del diedro formado por una cara triangular y otra decagonal, ambas regulares.

Aplicando la fórmula general [4], (ver lám. 33)

$$\boxed{\varphi_{3-10}} = \alpha_3 + \alpha_{10} = 84^\circ 20' 24,4'' + 58^\circ 16' 57,1'' =$$

$$= \boxed{142^\circ 37' 21,5''}$$

Puede obtenerse directamente de la siguiente manera:

$$\boxed{t_f \varphi_{3-10}} = t_f (\alpha_3 + \alpha_{10}) = \frac{t_f \alpha_3 + t_f \alpha_{10}}{1 - t_f \alpha_3 \times t_f \alpha_{10}} = \frac{\frac{5\sqrt{5}+9}{2} + \frac{\sqrt{5}+1}{2}}{1 - \frac{5\sqrt{5}+9}{2} \times \frac{\sqrt{5}+1}{2}} =$$

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$$= \frac{\frac{6\sqrt{5} + 10}{2}}{1 - \frac{25 + 9\sqrt{5} + 5\sqrt{5} + 9}{4}} = \frac{3\sqrt{5} + 5}{1 - \frac{34 + 14\sqrt{5}}{4}} = \frac{3\sqrt{5} + 5}{\frac{4 - 34 - 14\sqrt{5}}{4}} = \frac{4(3\sqrt{5} + 5)}{-30 - 14\sqrt{5}} =$$

$$= -\frac{2(3\sqrt{5} + 5)}{15 + 7\sqrt{5}} = -\frac{2(3\sqrt{5} + 5)(7\sqrt{5} - 15)}{49 \times 5 - 15^2} = -\frac{2(105 + 35\sqrt{5} - 45\sqrt{5} - 75)}{20} =$$

$$= -\frac{30 - 10\sqrt{5}}{10} = \boxed{-(3 - \sqrt{5})} = -0,76 \ 39 \ 32 \ 02 \dots$$

y haciendo  $\alpha_0 = \pi - \varphi_{3-10}$ , será  $\tan \alpha_0 = -\tan \varphi_{3-10} =$

$$= -(- (3 - \sqrt{5})) = 3 - \sqrt{5} = 0,76 \ 39 \ 32 \ 02 \dots \text{ de donde}$$

$$\tan \alpha_0 = 0,76 \ 39 \ 32 \ 02 \dots \quad \alpha_0 = 37^\circ \ 22' \ 38,5''$$

$$\boxed{\varphi_{3-10}} = 180^\circ - 37^\circ \ 22' \ 38,5'' = \boxed{142^\circ \ 37' \ 21,5''}$$

valor coincidente con el ya obtenido.

Ángulo rectilíneo " $\varphi_{10-10}$ " del diedro formado por dos caras diagonales.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{10-10}} = \alpha_{10} + \alpha_{10} = 2\alpha_{10} = 2 \times (58^\circ \ 16' \ 57,1'') =$$

$$= \boxed{116^\circ \ 33' \ 54,2''}$$

Puede obtenerse directamente, su tangente, de la siguiente manera:

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$$\begin{aligned} \boxed{\tan \frac{\varphi_{10-10}}{2}} &= \tan 2\alpha_0 = \frac{2 \tan \alpha_0}{1 - \tan^2 \alpha_0} = \frac{2 \times \frac{\sqrt{5}+1}{2}}{1 - \left(\frac{\sqrt{5}+1}{2}\right)^2} = \frac{\sqrt{5}+1}{1 - \frac{5+1+2\sqrt{5}}{4}} = \\ &= \frac{\sqrt{5}+1}{\frac{4-6-2\sqrt{5}}{4}} = \frac{4(\sqrt{5}+1)}{-2-2\sqrt{5}} = -\frac{4(\sqrt{5}+1)}{2(\sqrt{5}+1)} = \boxed{-2} \end{aligned}$$

y haciendo  $\alpha_0 = \pi - \varphi_{10-10}$ , será

$$\tan \alpha_0 = -\tan \varphi_{10-10} = -(-2) = 2$$

$$\tan \frac{\varphi_{10-10}}{2} = \tan 2 = 0.3010300 \quad \alpha_0 = 63^\circ 26' 5.8''$$

$$\begin{aligned} \text{de donde } \boxed{\varphi_{10-10}} &= 180^\circ - 63^\circ 26' 5.8'' = \\ &= \boxed{116^\circ 33' 54.2''} \end{aligned}$$

valor coincidente con el ya obtenido.

NOTA. - El valor del diedro  $\varphi_{10-10}$  es el mismo que el del dodecaedro regular (ver lám. 4, fórm. 34) lo cual nos indica que este arquimedianos puede obtenerse de dicho dodecaedro correspondiendo las caras pentagonales de éste con las decagonales de aquél.

### Área lateral "S" del arquimedianos

Se compone de la suma de 20 caras triangulares y 12 caras decagonales, ambas regulares y de igual lado "l"

--	--	--

1.  $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$   
 2.  $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$   
 3.  $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$

4.  $\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$   
 5.  $\frac{5}{8} \times \frac{3}{10} = \frac{15}{80} = \frac{3}{16}$   
 6.  $\frac{7}{9} \times \frac{2}{5} = \frac{14}{45}$

7.  $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$   
 8.  $\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$

9.  $\frac{3}{7} \times \frac{5}{8} = \frac{15}{56}$   
 10.  $\frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$   
 11.  $\frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$   
 12.  $\frac{6}{7} \times \frac{3}{4} = \frac{18}{28} = \frac{9}{14}$   
 13.  $\frac{7}{8} \times \frac{2}{9} = \frac{14}{72} = \frac{7}{36}$   
 14.  $\frac{8}{9} \times \frac{1}{3} = \frac{8}{27}$

15.  $\frac{9}{10} \times \frac{4}{5} = \frac{36}{50} = \frac{18}{25}$

16.  $\frac{10}{11} \times \frac{5}{6} = \frac{50}{66} = \frac{25}{33}$

17.  $\frac{11}{12} \times \frac{3}{4} = \frac{33}{48} = \frac{11}{16}$   
 18.  $\frac{12}{13} \times \frac{2}{3} = \frac{24}{39} = \frac{8}{13}$

La apotema "k" de un decágono regular de lado "l" y radio "d<sub>10</sub>" de su circunferencia circunscrita, será:

$$k = \sqrt{(d_{10})^2 - \left(\frac{l}{2}\right)^2} = \sqrt{\left(\frac{\sqrt{5}+1}{2} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{(\sqrt{5}+1)^2}{4} - \frac{1}{4}} \cdot l =$$

$$= \sqrt{\frac{5+1+2\sqrt{5}}{4} - \frac{1}{4}} \cdot l = \sqrt{\frac{6+2\sqrt{5}-1}{4}} \cdot l = \frac{\sqrt{5+2\sqrt{5}}}{2} \cdot l.$$

y el área lateral "S", será:

$$S = 20 \times \frac{\sqrt{3}}{4} l^2 + 12 \times \frac{10 l}{2} \times \frac{\sqrt{5+2\sqrt{5}}}{2} \cdot l = \boxed{(5\sqrt{3} + 30\sqrt{5+2\sqrt{5}}) l^2} =$$

$$= (8,66\ 02\ 54\ 04... + 92,33\ 05\ 05-90) l^2 = \boxed{100,99\ 07\ 59-94... l^2}$$

### Volumen "V" del arquimediano

Se compone de la suma de 20 pirámides triangulares regulares de lado "l" y altura "C<sub>3</sub>", y de 12 pirámides decagonales de lado "l" y altura "C<sub>10</sub>". Su volumen será pues:

$$\boxed{V} = 5\sqrt{3} l^2 \times \frac{C_3}{3} + 30 \sqrt{5+2\sqrt{5}} l^2 \times \frac{C_{10}}{3} =$$

$$= \frac{5\sqrt{3}}{3} \times \frac{5\sqrt{15} + 9\sqrt{3}}{12} l^3 + 10 \sqrt{5+2\sqrt{5}} \times \sqrt{\frac{25+11\sqrt{5}}{8}} \cdot l^3 =$$

$$= \left[ \frac{5}{36} (15\sqrt{5} + 27) + 10 \sqrt{\frac{(25+11\sqrt{5})(5+2\sqrt{5})}{8}} \right] \cdot l^3 = \left[ \frac{5(5\sqrt{5} + 9)}{12} + \right.$$

Date	Particulars	Amount
1870	To Balance	100.00
1871	By Cash	50.00
1872	To Cash	25.00
1873	By Cash	75.00
1874	To Cash	100.00
1875	By Cash	150.00
1876	To Cash	200.00
1877	By Cash	250.00
1878	To Cash	300.00

$$\begin{aligned}
 & + 5 \sqrt{\frac{125 + 55\sqrt{5} + 50\sqrt{5} + 110}{2}} \times l^3 = \left[ \frac{25\sqrt{5} + 45}{12} + 5 \sqrt{\frac{325 + 105\sqrt{5}}{2}} \right] \times l^3 = \\
 & = \left[ \frac{25\sqrt{5} + 45}{12} + 5 \sqrt{\frac{5(47 + 21\sqrt{5})}{2}} \right] \times l^3 = \left( \frac{25\sqrt{5} + 45}{12} + 5 \sqrt{\frac{5}{2}} \times \sqrt{47 + 21\sqrt{5}} \right) l^3 = \\
 & = \left( \frac{25\sqrt{5} + 45}{12} + 5 \times \sqrt{\frac{5}{2}} \left( \sqrt{\frac{49}{2}} + \sqrt{\frac{45}{2}} \right) \right) \times l^3 = \left( \frac{25\sqrt{5} + 45}{12} + 5 \left( \sqrt{\frac{5 \times 7^2}{2^2}} + \sqrt{\frac{5 \times 3^2}{2^2}} \right) \right) \times l^3 = \\
 & = \left( \frac{25\sqrt{5} + 45}{12} + \frac{35\sqrt{5}}{2} + \frac{75}{2} \right) \times l^3 = \left( \frac{25\sqrt{5} + 45 + 210\sqrt{5} + 450}{12} \right) \times l^3 = \\
 & = \boxed{\frac{495 + 235\sqrt{5}}{12}} l^3 = 85,03966456 \dots l^3
 \end{aligned}$$

FIGURA CORPÓREA

Se obtiene por el acoplamiento de 20 triángulos equiláteros, de lado  $l = 18,5 \text{ mm}$ , y 12 dodecaedros regulares de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren 2 dodecaedros y un triángulo.

En el cuadro sinóptico que damos a continuación, están resumidos los resultados analíticos obtenidos anteriormente.





## CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\sqrt{\frac{37 + 15\sqrt{5}}{8}} \ell$	2,96 94 49... $\ell$
$b$	$\frac{3\sqrt{5} + 5}{4} \ell$	2,92 70 51... $\ell$
$c_3$	$\frac{5\sqrt{5} + 9\sqrt{3}}{12} \ell$	2,91 27 81... $\ell$
$c_{10}$	$\sqrt{\frac{25 + 11\sqrt{5}}{8}} \ell$	2,48 98 98... $\ell$
$d_3$	$\frac{\sqrt{3}}{3} \ell$	0,57 73 51... $\ell$
$d_{10}$	$\frac{\sqrt{5} + 1}{2} \ell$	1,61 80 34... $\ell$
$m$	$\sqrt{\frac{5 \cdot (17 + 3\sqrt{5})}{122}} \ell$	0,98 57 22... $\ell$
$\alpha_3$	$\frac{1}{\ell} \alpha_3 = \frac{5\sqrt{5} + 9}{2}$	$\text{tg. } \alpha_3 = 10,09\ 01\ 70$ $\alpha_3 = 84^\circ\ 20'\ 24,4''$
$\alpha_{10}$	$\text{tg } \alpha_{10} = \frac{\sqrt{5} + 1}{2}$	$\text{tg } \alpha_{10} = 1,61\ 80\ 34$ $\alpha_{10} = 58^\circ\ 16'\ 57,1''$
$\varphi_{3-10}$	$\text{tg } \varphi_{3-10} = -(3 - \sqrt{5})$	$\text{tg } \varphi_{3-10} = -0,76\ 39\ 32$ $\varphi_{3-10} = 142^\circ\ 37'\ 21,5''$
$\varphi_{10-10}$	$\text{tg } \varphi_{10-10} = -2$	$\varphi_{10-10} = 116^\circ\ 33'\ 54,2''$
$S$	$(5\sqrt{3} + 30\sqrt{5 + 2\sqrt{5}}) \ell^2$	100,99 07 60... $\ell^2$
$V$	$\frac{495 + 235\sqrt{5}}{12} \ell^3$	85,03 96 65... $\ell^3$



Year	Month	Day	Time	Place	Remarks
1900	Jan	1	10:00	St. Paul	Arrived
1900	Jan	2	10:00	St. Paul	Left
1900	Jan	3	10:00	St. Paul	Arrived
1900	Jan	4	10:00	St. Paul	Left
1900	Jan	5	10:00	St. Paul	Arrived
1900	Jan	6	10:00	St. Paul	Left
1900	Jan	7	10:00	St. Paul	Arrived
1900	Jan	8	10:00	St. Paul	Left
1900	Jan	9	10:00	St. Paul	Arrived
1900	Jan	10	10:00	St. Paul	Left
1900	Jan	11	10:00	St. Paul	Arrived
1900	Jan	12	10:00	St. Paul	Left
1900	Jan	13	10:00	St. Paul	Arrived
1900	Jan	14	10:00	St. Paul	Left
1900	Jan	15	10:00	St. Paul	Arrived
1900	Jan	16	10:00	St. Paul	Left
1900	Jan	17	10:00	St. Paul	Arrived
1900	Jan	18	10:00	St. Paul	Left
1900	Jan	19	10:00	St. Paul	Arrived
1900	Jan	20	10:00	St. Paul	Left
1900	Jan	21	10:00	St. Paul	Arrived
1900	Jan	22	10:00	St. Paul	Left
1900	Jan	23	10:00	St. Paul	Arrived
1900	Jan	24	10:00	St. Paul	Left
1900	Jan	25	10:00	St. Paul	Arrived
1900	Jan	26	10:00	St. Paul	Left
1900	Jan	27	10:00	St. Paul	Arrived
1900	Jan	28	10:00	St. Paul	Left
1900	Jan	29	10:00	St. Paul	Arrived
1900	Jan	30	10:00	St. Paul	Left
1900	Jan	31	10:00	St. Paul	Arrived

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder en la lámina 41, a la representación gráfica del Arquimediano IX.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, de procesos gráficos y de cotas complementarias, cuyo cálculo efectuaremos posteriormente. Todas las magnitudes las obtendremos en función del lado " $l_{IX}$ " del arquimedianos, cuya longitud es de 18,5 mm.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_{IX} = \text{Dato del ejercicio} = 18,5 \text{ mm}$$

$$a = 2,969449... \times 18,5 = 55,0 \text{ mm}$$

$$b = 2,927051... \times 18,5 = 54,2 \text{ mm}$$

$$c_3 = 2,912781... \times 18,5 = 54,0 \text{ mm}$$

$$c_{10} = 2,489898... \times 18,5 = 46,1 \text{ mm}$$

$$d_3 = 0,577351... \times 18,5 = 10,7 \text{ mm}$$

$$d_6 = 1,618034... \times 18,5 = 30,0 \text{ mm}$$

Antes de proceder al trazado gráfico, observemos en la lámina 41 que la proyección del arquimedianos en el plano II, presenta una forma muy regular que facilita notablemente su trazado.

Las propiedades geométricas de ella, son:

- 1) Las caras 1 al 10 y 51 al 60, son dodecágonos regulares coincidentes, de lado " $l$ " y radio " $d_{10}$ " de su

My dear Sir,

I have the pleasure to inform you that the same has been forwarded to you by the post of the 10th inst. and I am sure it will be found to contain all the information you require.

I am, Sir, very respectfully,  
Your obedient servant,

- Yours faithfully,  
J. H. [Signature]
- Enclosed for you are the following documents:
- 1. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 2. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 3. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 4. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 5. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 6. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 7. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 8. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 9. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.
  - 10. A copy of the report of the Committee on the subject of the proposed amendment to the Bill.

I am, Sir, very respectfully,  
Your obedient servant,  
J. H. [Signature]



circunferencia circunscrita.

- 2) Los vértices 11 al 15 y 46 al 50, lo son de otro decágono regular de vértices alternados con el anterior; el radio de su circunferencia circunscrita es el " $r_1$ ".
- 3) Los vértices 16 al 20 y 41 al 45 también lo son de un decágono regular, de lados paralelos al anterior; el radio de su circunferencia circunscrita es el " $r_2$ ".
- 4) Los vértices 21 al 30 y 31 al 40 forman un polígono, no regular, que es el contorno aparente de la proyección sobre II del arquimedeano. Dicho polígono está inscrito en una circunferencia de radio " $r_3$ ". Los lados 22-23, 24-25, 26-27, 28-29, 30-21, así como los 32-33, 34-35, 36-37, 38-39 y 40-31, tienen la longitud " $l$ " de la arista, por ser todas paralelas a II; las rectas que unen sus puntos medios con el centro O, son coincidentes con las que unen los vértices 16 al 20 y 41 al 45, con el mismo centro.

Las propiedades anteriores permiten el trazado inmediato de la proyección total del arquimedeano sobre el plano II, calculando previamente los radios " $r_1$ ", " $r_2$ " y " $r_3$ ".

Teniendo presente lo expuesto, el orden de operaciones del trazado gráfico (lam. 41), es el siguiente:



1° Situar el centro  $O$ , de coordenadas 72, 72, 85 mm.

2° Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio 55 mm.

3° Representar en I, II y III las caras decagonales opuestas 1 al 10 y 51 al 60, supuesto el poliedro colocado con dichas caras paralelas a II y uno de sus lados (1-2 en la superior y 51-52 en la inferior) perpendiculares a I. Para obtener las proyecciones de ambas en II (son coincidentes), dibujar la circunferencia circunscrita de radio " $r_{10}$ " y dividirla en 10 partes iguales; uniendo los puntos de división se obtendrá un decágono regular de lado " $l$ " (comprobación). Las proyecciones de dichas caras en I y III son rectas paralelas al eje X equidistantes de  $O$  la magnitud " $C_{10}$ "; la magnitud del segmento se deduce de II (son diferentes en I y III)

4° Completar en II, la proyección del arquimediano, y proceder (teniendo en cuenta las propiedades enunciadas al principio) y proceder como sigue:

a) Trazar una circunferencia de radio " $r_1$ " (y centro  $O_{II}$ ) y dividirla en 10 partes iguales, tomando como origen el vértice 48; estos puntos de división nos darán los vértices 11 al 15 y 46 al 50.

b) Trazar una segunda circunferencia, de igual centro y radio " $r_2$ ", que se dividirá también en 10 partes iguales





tomando como origen el vértice 42; estos puntos de división nos darán los vértices 16 al 20 y 41 al 45.

c) Trazar otra circunferencia de centro  $O_{II}$  y radio " $r_3$ "; unir los vértices 16 al 20 y 41 al 45, con el centro  $O$  (estos radios pasarán por los vértices ya obtenidos 11 al 15 y 46 al 50). Trazar paralelas a ambos lados de cada radio, a la distancia  $\frac{r}{2}$ , que al cortar a la circunferencia anterior nos determinarán los restantes vértices de esta proyección, n° 21 al 30 y 31 al 40.

5° obtenida la proyección en II, completaremos la de I, trazando previamente por  $O_1$  y equidistantes de él, paralelas hacia arriba y hacia abajo, a las distancias  $f_1:2$ ,  $f_2:2$  y  $f_3:2$ , sobre las que se encontrarán respectivamente las proyecciones de los vértices 11 al 15, 46 al 50 en las primeras, 16 al 20, 41 al 45 en las segundas, y 21 al 30, 31 al 40 en las terceras. La posición de estos vértices se obtiene de la II. (comprobar que los vértices 12, 18, 42, 48 están sobre el contorno aparente de la esfera circunscrita).

6° Completar la proyección en III, auxiliándose de las obtenidas en I y II.

Como comprobación y necesaria ayuda para el trazado gráfico dado anteriormente, vamos a determinar analíticamente las siguientes magnitudes complementarias que





darán mayor exactitud a dicho trazado.

Altura "n" de una cara triangular

Se demuestra en Geometría, es

$$n = \frac{\sqrt{3}}{2} l = 0.8660254... l$$

Apotema "k" de una cara decagonal

su valor, deducido anteriormente (ver pág. 12), es

$$k = \frac{\sqrt{5 + 2\sqrt{5}}}{2} l = 1.53884176... l$$

Distancia "g," de los vértices 11 al 15 al plano de la cara decagonal 1 al 10, y de los vértices 46 al 50 a la cara decagonal 51 al 60

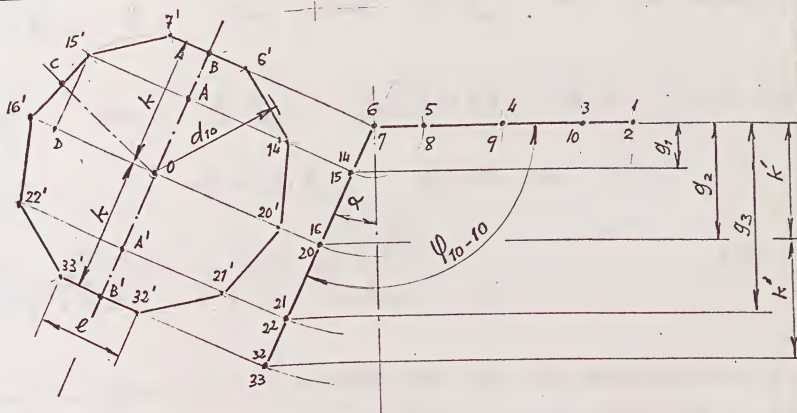


Figura 2

The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$

It is well known that this function is the arctangent function, i.e.  $f(x) = \arctan x$ . The function is odd, i.e.  $f(-x) = -f(x)$ , and it is bounded on the real line.

In the second part, we consider the function  $g(x)$  defined by the equation

$$g(x) = \int_0^x \frac{t}{1+t^2} dt$$

This function is even, i.e.  $g(-x) = g(x)$ , and it is unbounded on the real line.

The third part of the paper is devoted to the study of the function  $h(x)$  defined by the equation

$$h(x) = \int_0^x \frac{1}{1+t^4} dt$$

This function is odd, i.e.  $h(-x) = -h(x)$ , and it is bounded on the real line.



Consideremos (fig. 2) la cara superior decagonal 1 al 10 y la contigua (también decagonal) 6-14-20-21-32-33-22-16-15-7, proyectadas ambas sobre el plano I (ver lám. 41). Por ser los planos de ambas, perpendiculares a I, sus proyecciones serán segmentos rectilíneos que formen entre sí un ángulo  $\varphi_{10-10}$ , que es el del diedro de ambas caras. Supongamos rebatida sobre el plano del dibujo la cara 6-14...15-7, tomando como charnela la proyección rectilínea, con lo cual obtendremos la verdadera magnitud 6'-14'...15'-7' de dicho polígono.

El proceso a seguir para determinar las magnitudes " $g_1$ ", " $g_2$ " y " $g_3$ " consiste en proyectar dos segmentos correspondientes  $\overline{BA}$ ,  $\overline{BO}$  y  $\overline{BA'}$  sobre el plano III, siendo " $\alpha$ " el ángulo de proyección, cuyo valor es

$$\alpha = \varphi_{10-10} - \frac{\pi}{2} \quad \text{pero siendo} \quad \frac{1}{2} \varphi_{10-10} = -2 \quad (\text{ver estudio}$$

anterior), será  $\frac{1}{2} \varphi_{10-10} = \frac{1}{2} \left( \frac{\pi}{2} + \alpha \right) = -\operatorname{ctg} \alpha$  y por consiguiente

$$\operatorname{ctg} \alpha = 2, \quad \text{por lo que será}$$

$$\boxed{\cos \alpha} = \frac{\operatorname{ctg} \alpha}{\sqrt{1 + \operatorname{ctg}^2 \alpha}} = \frac{2}{\sqrt{1 + 2^2}} = \frac{2\sqrt{5}}{5} \quad [1]$$

Si en la figura 2, trazamos por O, la perpendicular al lado 15'-16', y por 15' la perpendicular al radio O-16', se nos formarán dos triángulos rectángulos

Date	Page	No.
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The first part of the paper is devoted to a general  
 introduction of the subject. It is shown that the  
 problem of the existence of a solution of the  
 differential equation  $y' = f(x, y)$  is equivalent to  
 the problem of the existence of a continuous  
 function  $y(x)$  satisfying the initial condition  
 $y(x_0) = y_0$  and the differential equation  
 $y' = f(x, y)$  in the interval  $(x_0 - \delta, x_0 + \delta)$ .

In the second part of the paper, the author  
 considers the case where  $f(x, y)$  is a  
 continuous function of  $x$  and  $y$  in the  
 interval  $(x_0 - \delta, x_0 + \delta) \times (y_0 - \delta, y_0 + \delta)$ .

It is shown that in this case there exists a  
 unique solution of the differential equation  
 satisfying the initial condition.

$$y(x) = y_0 + \int_{x_0}^x f(t, y(t)) dt$$

The third part of the paper is devoted to a  
 study of the case where  $f(x, y)$  is not  
 continuous. It is shown that in this case  
 there may or may not exist a solution.



$O-C-16'$  y  $15'-D-16'$  que son semejantes (tienen un ángulo agudo común), por lo que se verificará que

$$\frac{\overline{OC}}{O-16'} = \frac{\overline{15'-D}}{15'-16'} \quad \text{de donde} \quad 15'-D = \boxed{\overline{AO}} = \frac{(15'-16') \times \overline{OC}}{(O-16')} =$$

$$= \frac{l \times k}{d_{10}} = \left( \frac{\sqrt{5+2\sqrt{5}}}{2} l : \frac{\sqrt{5+1}}{2} l \right) \times l = \frac{\sqrt{5+2\sqrt{5}}}{\sqrt{5+1}} \times l =$$

$$= \frac{\sqrt{5+2\sqrt{5}} \times (\sqrt{5}-1)}{4} l = \frac{\sqrt{(5+2\sqrt{5})(\sqrt{5}-1)^2}}{4} l = \sqrt{\frac{(5+2\sqrt{5})(6-2\sqrt{5})}{16}} \times l =$$

$$= \sqrt{\frac{(5+2\sqrt{5})(3-\sqrt{5})}{8}} \times l = \sqrt{\frac{15+6\sqrt{5}-5\sqrt{5}-10}{8}} l = \boxed{\sqrt{\frac{5+\sqrt{5}}{8}} l}$$

De la figura 2, se deduce que

$$\boxed{\overline{BA}} = \overline{BO} - \overline{AO} = k - \overline{AO} = \frac{\sqrt{5+2\sqrt{5}}}{2} l - \sqrt{\frac{5+\sqrt{5}}{8}} l = \left( \frac{\sqrt{5+2\sqrt{5}}}{2} - \sqrt{\frac{5+\sqrt{5}}{8}} \right) l =$$

$$= \sqrt{\left( \frac{\sqrt{5+2\sqrt{5}}}{2} - \sqrt{\frac{5+\sqrt{5}}{8}} \right)^2} \times l = \sqrt{\frac{5+2\sqrt{5}}{4} + \frac{5+\sqrt{5}}{8} - 2 \times \frac{\sqrt{5+2\sqrt{5}}}{2} \times \sqrt{\frac{5+\sqrt{5}}{8}}} \times l =$$

$$= \sqrt{\frac{10+4\sqrt{5}+5+\sqrt{5}}{8} - \frac{\sqrt{(5+2\sqrt{5})(5+\sqrt{5})}}{8}} \times l = \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{25+10\sqrt{5}+5\sqrt{5}+10}}{8}} \times l =$$

$$= \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{35+15\sqrt{5}}}{8}} \times l = \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{5(7+3\sqrt{5})}}{8}} \times l =$$

$$= \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{5}}{8} \times \sqrt{7+3\sqrt{5}}} \times l = \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{5}}{8} \times \left( \sqrt{\frac{7}{2}} + \sqrt{\frac{5}{2}} \right)} \times l =$$

$$= \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{\sqrt{45}}{16} - \frac{\sqrt{25}}{16}} \times l = \sqrt{\frac{15+5\sqrt{5}}{8} - \frac{3\sqrt{5}}{4} - \frac{5}{4}} \times l =$$

Main body of the page containing faint, illegible text arranged in horizontal lines.

$$= \sqrt{\frac{15 + 5\sqrt{5} - 6\sqrt{5} - 10}{8}} \cdot l = \boxed{\sqrt{\frac{5 - \sqrt{5}}{8}} \cdot l}$$

De la misma figura 2, se deduce también que

$$\boxed{\overline{BA}'} = \overline{BO} + \overline{AO} = k + \overline{AO} = \frac{\sqrt{5} + 2\sqrt{5}}{2} l + \sqrt{\frac{5 + \sqrt{5}}{8}} l = (\text{siguiendo los mismos}$$

pasos de  $\overline{BA}$ , con el cambio de signo de  $-$  en  $+$ ) =

$$= \sqrt{\frac{15 + 5\sqrt{5}}{8}} + \frac{3\sqrt{5}}{4} + \frac{5}{4} \cdot l = \sqrt{\frac{15 + 5\sqrt{5} + 6\sqrt{5} + 10}{8}} \cdot l = \boxed{\sqrt{\frac{25 + 11\sqrt{5}}{8}} \cdot l}$$

Con los resultados anteriores, podremos obtener los valores de  
 "g<sub>1</sub>", "g<sub>2</sub>" y "g<sub>3</sub>", que serán respectivamente

$$\boxed{g_1} = \overline{BA} \times \cos \alpha = \sqrt{\frac{5 - \sqrt{5}}{8}} \cdot l \times \frac{2\sqrt{5}}{5} = \frac{2}{5} \times \sqrt{\frac{5(5 - \sqrt{5})}{8}} \cdot l = \sqrt{\frac{2^2 \times 5(5 - \sqrt{5})}{5^2 \times 8}} l =$$

$$= \boxed{\sqrt{\frac{5 - \sqrt{5}}{10}} \cdot l} = 0,52\ 57\ 31\ 1 \dots l$$

Para el caso del dibujo, será:  $g_1 = 0,52\ 57\ 31\ 1 \dots \times 18,5 = 9,7\ \text{mm}$

$$\boxed{g_2} = \overline{BO} \times \cos \alpha = k \times \cos \alpha = \frac{\sqrt{5} + 2\sqrt{5}}{2} l \times \frac{2\sqrt{5}}{5} = \frac{2 \times \sqrt{25 + 10\sqrt{5}}}{10} l =$$

$$= \frac{\sqrt{5 \cdot (5 + 2\sqrt{5})}}{5} l = \boxed{\sqrt{\frac{5 + 2\sqrt{5}}{5}} \cdot l} = 1,37\ 63\ 820 \dots l = 9,7\ \text{mm}$$

$$\boxed{g_3} = \overline{BA'} \times \cos \alpha = \sqrt{\frac{25 + 11\sqrt{5}}{8}} \cdot l \times \frac{2\sqrt{5}}{5} l = \frac{2}{5} \sqrt{\frac{125 + 55\sqrt{5}}{8}} \cdot l =$$

$$= \sqrt{\frac{4 \times 5 \times (25 + 11\sqrt{5})}{5^2 \times 8}} \cdot l = \boxed{\sqrt{\frac{25 + 11\sqrt{5}}{10}} \cdot l} = 2,22\ 70\ 32\ 73 \dots l$$

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations. The second part is devoted to the construction of the solution. The third part is devoted to the study of the properties of the solution. The fourth part is devoted to the application of the results to the theory of differential equations. The fifth part is devoted to the study of the properties of the solution. The sixth part is devoted to the application of the results to the theory of differential equations. The seventh part is devoted to the study of the properties of the solution. The eighth part is devoted to the application of the results to the theory of differential equations. The ninth part is devoted to the study of the properties of the solution. The tenth part is devoted to the application of the results to the theory of differential equations.

Distancia "f<sub>1</sub>" entre los dos planos paralelos a II, que contienen los vértices 11 al 15 y 16 al 50 respectivamente.

Se obtiene por diferencia de las alturas "C<sub>10</sub>" y "g<sub>1</sub>", ya calculadas.

$$\begin{aligned}
 \boxed{f_1} &= 2 (C_{10} - g_1) = 2 \times \left( \sqrt{\frac{25+11\sqrt{5}}{8}} l - \sqrt{\frac{5-\sqrt{5}}{10}} l \right) = \\
 &= 2 \left( \sqrt{\frac{25+11\sqrt{5}}{8}} - \sqrt{\frac{5-\sqrt{5}}{10}} \right) \times l = 2 \times \sqrt{\left( \sqrt{\frac{25+11\sqrt{5}}{8}} - \sqrt{\frac{5-\sqrt{5}}{10}} \right)^2} \times l = \\
 &= 2 \times \sqrt{\frac{25+11\sqrt{5}}{8} + \frac{5-\sqrt{5}}{10} - 2 \times \sqrt{\frac{25+11\sqrt{5}}{8}} \times \sqrt{\frac{5-\sqrt{5}}{10}}} \times l = \\
 &= 2 \times \sqrt{\frac{125+55\sqrt{5}+20-4\sqrt{5}}{40} - \frac{4(25+11\sqrt{5})(5-\sqrt{5})}{80}} \times l = \\
 &= 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \frac{125+55\sqrt{5}-25\sqrt{5}-55}{20}} \times l = 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \frac{70+30\sqrt{5}}{20}} \times l = \\
 &= 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \frac{\sqrt{7+3\sqrt{5}}}{\sqrt{2}}} \times l = 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \left( \sqrt{\frac{9}{2}} + \sqrt{\frac{5}{2}} \right) : \sqrt{2}} \times l = \\
 &= 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \sqrt{\frac{9}{4}} - \sqrt{\frac{5}{4}}} \times l = 2 \times \sqrt{\frac{145+51\sqrt{5}}{40} - \frac{3}{2} - \frac{\sqrt{5}}{2}} \times l = \\
 &= 2 \times \sqrt{\frac{145+51\sqrt{5}-60-20\sqrt{5}}{40}} \times l = \boxed{\sqrt{\frac{85+31\sqrt{5}}{10}}} \times l = 3,92833435... l
 \end{aligned}$$

Para el caso del dibujo, es:  $f_1 = 3,92833435... \times 18,53 = 72,8 \text{ mm}$



प्रमाणित किया जाता है कि श्री \_\_\_\_\_  
राज्य \_\_\_\_\_ के निवासी हैं।  
जो कि \_\_\_\_\_ के \_\_\_\_\_  
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Radio "r<sub>1</sub>" de la circunferencia circunscrita al decágono regular de vértices 11 al 15 y 46 al 50.

Este radio es un cateto de un triángulo rectángulo de hipotenusa "a" y el otro cateto " $\frac{f_1}{2}$ ". Su valor será:

$$\begin{aligned} r_1 &= \sqrt{a^2 - \left(\frac{f_1}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{37+15\sqrt{5}}{8}} \ell\right)^2 - \left(\frac{1}{2} \sqrt{\frac{85+31\sqrt{5}}{10}} \ell\right)^2} = \\ &= \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{1}{4} \times \frac{85+31\sqrt{5}}{10}} \times \ell = \sqrt{\frac{37+15\sqrt{5}}{8} - \frac{85+31\sqrt{5}}{40}} \times \ell = \\ &= \sqrt{\frac{185+75\sqrt{5}-85-31\sqrt{5}}{40}} \times \ell = \sqrt{\frac{100+44\sqrt{5}}{40}} \times \ell = \sqrt{\frac{25+11\sqrt{5}}{10}} \times \ell = \\ &= 2,22703273... \ell \end{aligned}$$

Para el caso del dibujo, será:  $r_1 = 2,22703273... \times 18,53 = 41,2 \text{ mm}$

Distancia "g<sub>2</sub>" de los vértices 16 al 20 al plano de la cara decagonal 1 al 10, y de los vértices 41 al 45 al de la cara decagonal 51 al 60

Esta magnitud ha sido ya determinada en el cálculo de "g<sub>1</sub>" (ver hoja 22); su valor es

$$g_2 = \sqrt{\frac{5+2\sqrt{5}}{5}} \ell = 1,3763820... \times \ell$$

Para el caso del dibujo, será:  $g_2 = 1,3763820... \times 18,53 = 25,5 \text{ mm}$

The first part of the paper is devoted to a study of the  
 properties of the function  $f(x)$  defined by the equation  

$$f(x) = \int_0^x \frac{1}{1+t^2} dt$$
 which is known to be the arctangent function.

In the second part we consider the function  $F(x)$  defined by  

$$F(x) = \int_0^x \frac{1}{1+t^2} dt$$
 and show that it is a solution of the differential equation  

$$F'(x) = \frac{1}{1+x^2}$$
 with the initial condition  $F(0) = 0$ .

The third part of the paper is devoted to a study of the  
 properties of the function  $G(x)$  defined by the equation  

$$G(x) = \int_0^x \frac{1}{1+t^2} dt$$
 and show that it is a solution of the differential equation  

$$G'(x) = \frac{1}{1+x^2}$$
 with the initial condition  $G(0) = 0$ .

In the fourth part we consider the function  $H(x)$  defined by  

$$H(x) = \int_0^x \frac{1}{1+t^2} dt$$
 and show that it is a solution of the differential equation  

$$H'(x) = \frac{1}{1+x^2}$$
 with the initial condition  $H(0) = 0$ .

The fifth part of the paper is devoted to a study of the  
 properties of the function  $I(x)$  defined by the equation  

$$I(x) = \int_0^x \frac{1}{1+t^2} dt$$
 and show that it is a solution of the differential equation  

$$I'(x) = \frac{1}{1+x^2}$$
 with the initial condition  $I(0) = 0$ .

The sixth part of the paper is devoted to a study of the  
 properties of the function  $J(x)$  defined by the equation  

$$J(x) = \int_0^x \frac{1}{1+t^2} dt$$
 and show that it is a solution of the differential equation  

$$J'(x) = \frac{1}{1+x^2}$$
 with the initial condition  $J(0) = 0$ .

Distancia " $f_2$ " entre los planos paralelos a  $\Pi$ , que contienen los vértices 16 al 20 y 41 al 45 respectivamente.

Se obtiene por diferencia de las alturas " $c_{10}$ " y " $g_2$ ", ya calculadas.

$$\begin{aligned}
 \boxed{f_2} &= 2(c_{10} - g_2) = 2 \times \left( \sqrt{\frac{25 + 11\sqrt{5}}{8}} \cdot l - \sqrt{\frac{5 + 2\sqrt{5}}{5}} \cdot l \right) = \\
 &= 2 \times \left( \sqrt{\frac{25 + 11\sqrt{5}}{8}} - \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right) \cdot l = 2 \times \sqrt{\left( \sqrt{\frac{25 + 11\sqrt{5}}{8}} - \sqrt{\frac{5 + 2\sqrt{5}}{5}} \right)^2} \cdot l = \\
 &= 2 \sqrt{\frac{25 + 11\sqrt{5}}{8} + \frac{5 + 2\sqrt{5}}{5} - 2 \sqrt{\frac{(25 + 11\sqrt{5})(5 + 2\sqrt{5})}{40}}} \cdot l = \\
 &= 2 \sqrt{\frac{125 + 55\sqrt{5} + 40 + 16\sqrt{5}}{40} - 2 \sqrt{\frac{125 + 55\sqrt{5} + 50\sqrt{5} + 110}{40}}} \cdot l = \\
 &= 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - 2 \sqrt{\frac{235 + 105\sqrt{5}}{40}}} \cdot l = 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - 2 \sqrt{\frac{47 + 21\sqrt{5}}{8}}} \cdot l = \\
 &= 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - \sqrt{\frac{47 + 21\sqrt{5}}{2}}} \cdot l = 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - \frac{1}{\sqrt{2}} \cdot \sqrt{47 + 21\sqrt{5}}} \cdot l = \\
 &= 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - \frac{1}{\sqrt{2}} \cdot \left( \sqrt{\frac{49}{2}} + \sqrt{\frac{45}{2}} \right)} \cdot l = 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - \sqrt{\frac{49}{4}} - \sqrt{\frac{45}{4}}} \cdot l = \\
 &= 2 \sqrt{\frac{165 + 71\sqrt{5}}{40} - \frac{7}{2} - \frac{3\sqrt{5}}{2}} \cdot l = 2 \sqrt{\frac{165 + 71\sqrt{5} - 140 - 60\sqrt{5}}{40}} \cdot l = \\
 &= 2 \sqrt{\frac{25 + 11\sqrt{5}}{40}} \cdot l = \boxed{\sqrt{\frac{25 + 11\sqrt{5}}{10}}} \cdot l = 2,22\ 70\ 32\ 73 \dots l
 \end{aligned}$$

Para el caso del dibujo, sea:  $f_2 = 2,22\ 70\ 32\ 73 \dots l = 41,2\ \text{mm}$





Radio "r<sub>2</sub>" de la circunferencia circunscrita al decágono regular de vértices 16 al 20 y 41 al 45

Este radio es un cateto del triángulo rectángulo de hipotenusa "a" y el otro cateto " $\frac{f_2}{2}$ ". Su valor será:

$$\begin{aligned} \boxed{r_2} &= \sqrt{a^2 - \left(\frac{f_2}{2}\right)^2} = \sqrt{\left(\left(\frac{\sqrt{37+15\sqrt{5}}}{8} \cdot l\right)^2 - \left(\frac{1}{2} \sqrt{\frac{25+11\sqrt{5}}{10}} l\right)^2\right)} = \\ &= \sqrt{\frac{\sqrt{37+15\sqrt{5}}}{8} - \frac{1}{4} \times \frac{25+11\sqrt{5}}{10}} \times l = \sqrt{\frac{\sqrt{37+15\sqrt{5}}}{8} - \frac{25+11\sqrt{5}}{40}} \times l = \\ &= \sqrt{\frac{185+75\sqrt{5}-25-11\sqrt{5}}{40}} \times l = \sqrt{\frac{160+64\sqrt{5}}{40}} \times l = \boxed{\sqrt{\frac{20+8\sqrt{5}}{5}}} \times l = \\ &= 2,75\ 27\ 63\ 84\dots l \end{aligned}$$

Para el caso del dibujo, será:  $r_2 = 2,75\ 27\ 63\ 84\dots l \times 18,53 = 51,0\text{ mm}$

Distancia "g<sub>3</sub>" de los vértices 21 al 30 al plano de la cara decagonal 1 al 10, y de los vértices 31 al 40 al de la cara decagonal 51 al 60

Esta magnitud ha sido ya determinada en el cálculo de "g<sub>1</sub>" (ver hoja 22); su valor es

$$\boxed{g_3} = \sqrt{\frac{25+11\sqrt{5}}{10}} \times l = 2,22\ 70\ 32\ 73\dots$$

Para el caso del dibujo, será:  $g_3 = 2,22\ 70\ 32\ 73\dots \times 18,53 = 41,2\text{ mm}$

1. The first part of the document is a letter from the Secretary of the Department of Education to the Director of the Bureau of Education. The letter is dated 1st January 1900 and is addressed to the Director of the Bureau of Education, Washington, D.C.

2. The second part of the document is a report on the progress of the work of the Bureau of Education during the year 1899. The report is dated 1st January 1900 and is addressed to the Secretary of the Department of Education.

3. The third part of the document is a list of the names of the members of the Bureau of Education during the year 1899. The list is dated 1st January 1900 and is addressed to the Secretary of the Department of Education.

4. The fourth part of the document is a list of the names of the members of the Bureau of Education during the year 1899. The list is dated 1st January 1900 and is addressed to the Secretary of the Department of Education.

5. The fifth part of the document is a list of the names of the members of the Bureau of Education during the year 1899. The list is dated 1st January 1900 and is addressed to the Secretary of the Department of Education.

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8. The eighth part of the document is a list of the names of the members of the Bureau of Education during the year 1899. The list is dated 1st January 1900 and is addressed to the Secretary of the Department of Education.

Distancia " $f_3$ " entre los planos paralelos a II, que contienen los vértices 21 al 30 y 31 al 40 respectivamente.

Se obtiene por diferencias de las alturas " $c_{10}$ " y " $g_3$ ", ya calculadas.

$$\begin{aligned}
 \boxed{f_3} &= 2 (c_{10} - g_3) = 2 \cdot \left( \sqrt{\frac{25 + 11\sqrt{5}}{8}} - \sqrt{\frac{25 + 11\sqrt{5}}{10}} \right) l = \\
 &= 2 \cdot \sqrt{\left( \sqrt{\frac{25 + 11\sqrt{5}}{8}} - \sqrt{\frac{25 + 11\sqrt{5}}{10}} \right)^2} \cdot l = 2 \cdot \sqrt{\frac{25 + 11\sqrt{5}}{8} + \frac{25 + 11\sqrt{5}}{10} - 2 \sqrt{\frac{(25 + 11\sqrt{5})^2}{80}}} \cdot l = \\
 &= 2 \cdot \sqrt{\frac{125 + 55\sqrt{5} + 100 + 44\sqrt{5}}{40} - \frac{25 + 11\sqrt{5}}{\sqrt{20}}} \cdot l = 2 \cdot \sqrt{\frac{225 + 99\sqrt{5}}{40} - \frac{25 + 11\sqrt{5}}{2\sqrt{5}}} \cdot l = \\
 &= 2 \cdot \sqrt{\frac{225 + 99\sqrt{5}}{40} - \frac{25\sqrt{5} + 55}{10}} \cdot l = 2 \cdot \sqrt{\frac{225 + 99\sqrt{5} - 100\sqrt{5} - 220}{40}} \cdot l = \\
 &= \boxed{\sqrt{\frac{5 - \sqrt{5}}{10}}} \cdot l = 0,5257311... \cdot l
 \end{aligned}$$

Para el caso del dibujo, será:  $f_3 = 0,5257311... \times 18,53 = 9,8 \text{ m.m.}$

Radio " $r_3$ " de la circunferencia circunscrita al polígono no regular de 20 lados, de vértices 21 al 30 y 31 al 40

Este radio es un cateto del triángulo rectángulo de hipotenusa " $a$ " y el otro cateto " $\frac{f_3}{2}$ ". Su valor será:

$$\boxed{r_3} = \sqrt{a^2 - \left(\frac{f_3}{2}\right)^2} = \sqrt{\left(\sqrt{\frac{37 + 15\sqrt{5}}{8}} l\right)^2 - \left(\frac{1}{2} \cdot \sqrt{\frac{5 - \sqrt{5}}{10}} l\right)^2} =$$

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$$= \sqrt{\frac{37 + 15\sqrt{5}}{8}} - \frac{1}{4} \times \frac{5 - \sqrt{5}}{10} \times l = \sqrt{\frac{37 + 15\sqrt{5}}{8}} - \frac{5 - \sqrt{5}}{40} \times l = \sqrt{\frac{125 + 75\sqrt{5} - 5 - \sqrt{5}}{40}} \times l =$$

$$= \sqrt{\frac{120 + 74\sqrt{5}}{40}} \times l = \boxed{\sqrt{\frac{90 + 37\sqrt{5}}{20}}} \times l = 2,93 \ 88 \ 30 \ 68... l$$

Para el caso del dibujo, será  $r_3 = 2,93 \ 88 \ 30 \ 68... \times 18,53 = 54,5 \text{ m m.}$

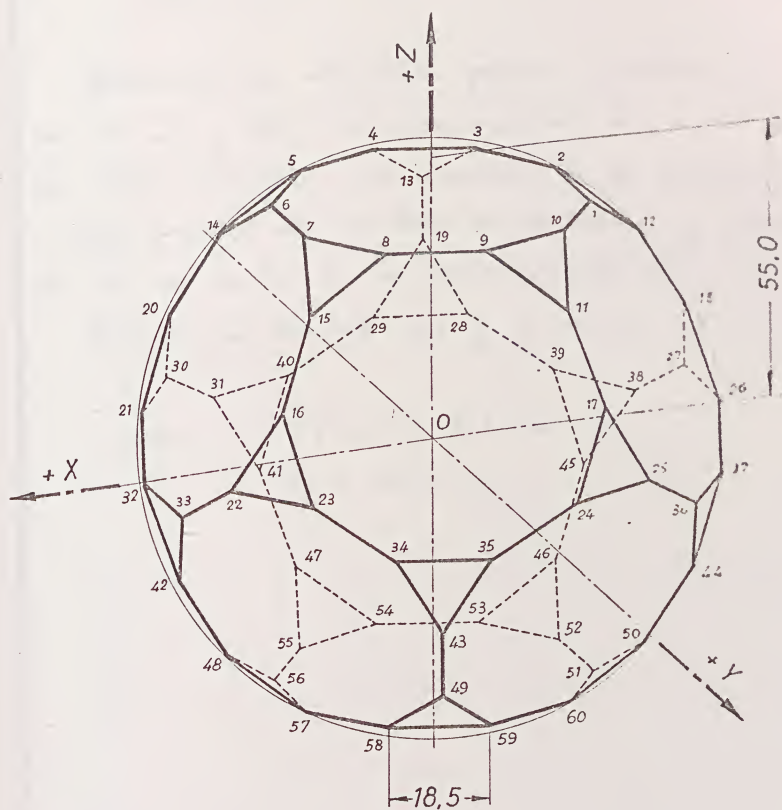
En el cuadro sinóptico que damos a continuación, resumimos los resultados de los valores complementarios deducidos

### CUADRO SINÓPTICO DE LAS MAGNITUDES COMPLEMENTARIAS

Magnitud	Valor exacto	Valor decimal aproximado
$n$	$\frac{\sqrt{3}}{2} l$	0,86 60 25... l
$k$	$\frac{\sqrt{5} + 2\sqrt{5}}{2} l$	1,53 88 42... l
$f_1$	$\sqrt{\frac{85 + 37\sqrt{5}}{10}} l$	3,92 83 34... l
$f_2$	$\sqrt{\frac{25 + 11\sqrt{5}}{10}} l$	2,22 70 33... l
$f_3$	$\sqrt{\frac{5 - \sqrt{5}}{10}} l$	0,52 57 31... l
$g_1$	$\sqrt{\frac{5 - \sqrt{5}}{10}} l$	0,52 57 31... l
$g_2$	$\sqrt{\frac{5 + 2\sqrt{5}}{5}} l$	1,37 63 82... l
$g_3$	$\sqrt{\frac{25 + 11\sqrt{5}}{10}} l$	2,22 70 33... l
$r_1$	$\sqrt{\frac{25 + 11\sqrt{5}}{10}} l$	2,22 70 33... l
$r_2$	$\sqrt{\frac{20 + 8\sqrt{5}}{5}} l$	2,75 27 64... l
$r_3$	$\sqrt{\frac{90 + 37\sqrt{5}}{20}} l$	2,93 88 31... l
Relaciones notables: $f_2 = g_3 = r_1$ $f_3 = g_1$		







Arquimediano IX



ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimedeano  $X$ , en el que en cada vértice concurren un cuadrado y dos hexágonos regulares. La longitud de su lado es de 34.8 mm, y las coordenadas de su centro  $O$ , son  $O(72, 72, 85)$  mm.

Dibujar en formato A3V y a escala 1:1

DATOS

 $O(72, 72, 85)$  mm $l_X = 34.8$  mm

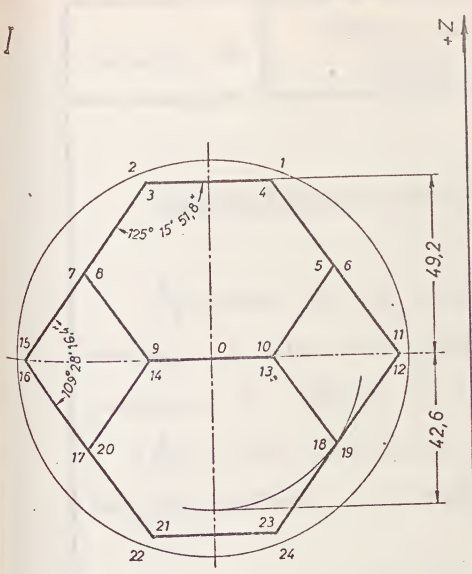
1900

Received of the Treasurer of the  
 Board of Directors of the  
 City of New York the sum of  
 \$100.00 for the year 1900  
 and for the year 1901  
 and for the year 1902  
 and for the year 1903  
 and for the year 1904  
 and for the year 1905  
 and for the year 1906  
 and for the year 1907  
 and for the year 1908  
 and for the year 1909  
 and for the year 1910

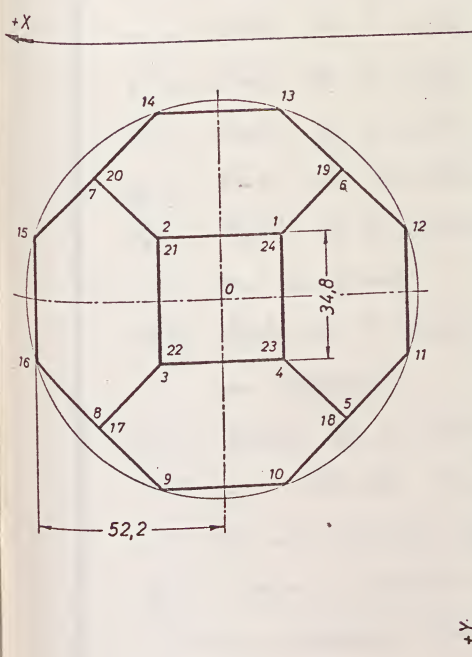
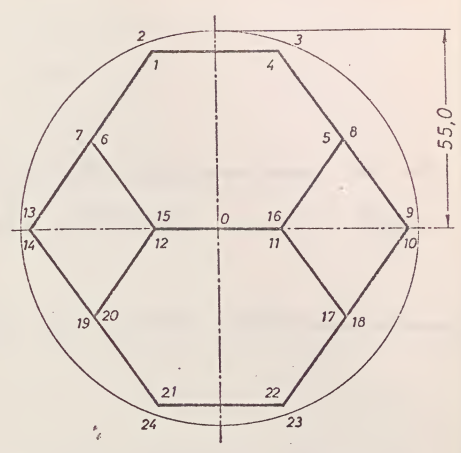
Witness my hand and seal  
 this 1st day of January 1900



I



III



ARQUIMEDIANO X

- Número de caras cuadradas.....  $C_4 = 6$
- Número de caras exagonales.....  $C_6 = 8$
- Número de vértices.....  $V = 24$
- Número de aristas.....  $A = 36$
- Número de caras de un ángulo sólido.....  $1P_4 + 2P_6$

ENUNCIADO

Representar por el método gráfico-analítico en los planos I, II y III, el Arquimediano X, en el que en cada vértice concurren un cuadrado y dos exágonos regulares.

La longitud de su lado es de 34,8 milímetros y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

II

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela Curso
Fecha:					
Alumno:					
Escala	Arquimediano X				Lámina 42
1:1					Curso 19 - 19



CONSIDERACIONES PREVIAS

Seguiremos en el estudio de este arquimedianos, las directrices y fórmulas generales planteadas en el "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

$l$  = Arista del Arquimediano X (dato del ejercicio).

$a$  = Radio de la esfera circunscrita

$b$  = Radio de la esfera tangente a las aristas.

$c_4$  = Radio de la esfera tangente a las caras cuadradas.

$c_6$  = Radio de la esfera tangente a las caras hexagonales.

$d_4$  = Radio de la circunferencia circunscrita a una cara cuadrada.

$d_6$  = Radio de la circunferencia circunscrita a una cara hexagonal.

$m$  = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

$\alpha_4$  = Ángulo rectilíneo del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\alpha_6$  = Ángulo rectilíneo del diedro formado por una



cara exagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\varphi_{4-6}$  = Ángulo rectilíneo del diedro formado por una cara triangular y otra octogonal.

$\varphi_{6-6}$  = Ángulo rectilíneo del diedro formado por dos caras exagonales.

$S$  = Superficie

$V$  = Volumen

### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimediano, nos indica que se compone de 6 caras cuadradas y 8 caras exagonales; 24 vértices y 36 aristas.

En cada vértice concurren un cuadrado y dos exágonos regulares.

Así pues, tendremos que:

$\text{ARQUIMEDIANO X } (1 P_4 + 2 P_6); C_4 = 6; C_6 = 8; V = 24; A = 36$
--

### Cálculo de sus magnitudes

Arista "l" del arquimediano

Dato del ejercicio



My dear Sir,  
I have the honor to acknowledge the receipt of your letter of the 10th inst. in relation to the matter of the ...  
I am sorry to hear that you are not satisfied with the result of the ...  
I have no objection to your making such use of the facts as you may think proper, but I must beg to say that I am not responsible for the ...  
I am, Sir, very respectfully,  
Your obedient servant,  
[Signature]

THE SECRETARY OF THE BOARD OF TRADE

I have the honor to acknowledge the receipt of your letter of the 10th inst. in relation to the matter of the ...  
I am sorry to hear that you are not satisfied with the result of the ...  
I have no objection to your making such use of the facts as you may think proper, but I must beg to say that I am not responsible for the ...  
I am, Sir, very respectfully,  
Your obedient servant,  
[Signature]

Very respectfully,  
[Signature]

I am, Sir, very respectfully,  
Your obedient servant,  
[Signature]

Radio "m" de la circunferencia circunscrita al polígono obtenido al unir los extremos de las tres aristas que concurren en un ángulo sólido.

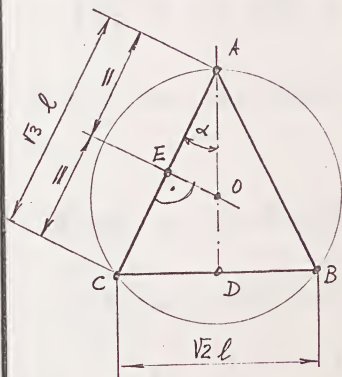


Figura 1

Dicho polígono (fig.1), es un triángulo isósceles, cuya base  $\overline{BC}$  es la diagonal de una cara cuadrada, y sus otros dos lados iguales  $AC = AB$ , corresponden a la diagonal de una cara hexagonal.

Se demuestra en Geometría que la diagonal de un cuadrado de lado "l", es

$$\overline{CB} = \sqrt{2} \, l$$

y la de un hexágono regular, también de lado "l", es

$$\overline{AC} = \overline{AB} = \sqrt{3} \, l$$

De la figura se deduce:

$$\overline{AD} = \sqrt{\overline{AC}^2 - \overline{CD}^2} = \sqrt{(\sqrt{3} \, l)^2 - \left(\frac{1}{2} \sqrt{2} \, l\right)^2} = \sqrt{3 - \frac{1}{2}} \times l = \sqrt{\frac{5}{2}} \times l$$

por lo que será:

$$\cos \alpha = \frac{\overline{AD}}{\overline{AC}} = \frac{\sqrt{\frac{5}{2}} \, l}{\sqrt{3} \, l} = \sqrt{\frac{5}{6}}$$

y en consecuencia:

The first part of the theory is the theory of the origin of the earth. It is a theory which is based on the fact that the earth is a sphere and that it is composed of different layers. The first layer is the crust, the second is the mantle, and the third is the core.

The second part of the theory is the theory of the origin of life. It is a theory which is based on the fact that life is a complex of different parts. The first part is the cell, the second is the tissue, the third is the organ, the fourth is the system, the fifth is the organism, the sixth is the population, the seventh is the community, the eighth is the society, the ninth is the state, the tenth is the nation, the eleventh is the world.



The third part of the theory is the theory of the origin of the human race. It is a theory which is based on the fact that the human race is a complex of different parts. The first part is the individual, the second is the family, the third is the tribe, the fourth is the nation, the fifth is the world.

The fourth part of the theory is the theory of the origin of the universe. It is a theory which is based on the fact that the universe is a complex of different parts. The first part is the atom, the second is the molecule, the third is the cell, the fourth is the tissue, the fifth is the organ, the sixth is the system, the seventh is the organism, the eighth is the population, the ninth is the community, the tenth is the society, the eleventh is the state, the twelfth is the nation, the thirteenth is the world.

$$A0 = \boxed{m} = \frac{\overline{AE}}{\cos \alpha} = \frac{\sqrt{3}}{2} l : \sqrt{\frac{5}{6}} = \frac{1}{2} \sqrt{3 \cdot \frac{5}{6}} \cdot l = \frac{1}{2} \sqrt{\frac{18}{5}} \cdot l =$$

$$= \frac{3}{2} \sqrt{\frac{2}{5}} l = \frac{3}{2} \sqrt{\frac{10}{25}} l = \boxed{\frac{3\sqrt{10}}{10}} l = 0,94868330... l$$

Para el caso del dibujo, será:  $m = 0,9486833... \times 34,79 = 33,0 \text{ mm}$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [1] (ver lám. 33)

$$\boxed{a} = \frac{l^2}{2\sqrt{l^2 - m^2}} = \frac{l^2}{2\sqrt{l^2 - \left(\frac{3\sqrt{10}}{10} l\right)^2}} = \frac{1}{2\sqrt{1 - \frac{90}{100}}} l = \frac{1}{2\sqrt{1 - \frac{9}{10}}} \cdot l =$$

$$= \frac{1}{2\sqrt{\frac{1}{10}}} \cdot l = \frac{1}{2} \sqrt{10} l = \boxed{\frac{\sqrt{10}}{2}} l = 1,58113883... l$$

Para el caso del dibujo, será:  $a = 55 \text{ mm}$   $l = 34,79 \text{ mm}$

Radio "b" de la esfera tangente a las aristas

Se obtiene aplicando la fórmula general [3] (ver lám. 33)

$$\boxed{b} = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{10}}{2} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{10}{4} - \frac{1}{4}} l = \sqrt{\frac{9}{4}} \cdot l = \boxed{\frac{3}{2}} l =$$

$$= 1,5 \cdot l$$

Para el caso del dibujo, será:  $b = 1,5 \times 34,79 = 52,2 \text{ mm}$

1871-1872

1873-1874

1875-1876

1877-1878

1879-1880

1881-1882

1883-1884

1885-1886

1887-1888

1889-1890

1891-1892

1893-1894



Radio " $d_4$ " de la circunferencia circunscrita a una cara cuadrada de lado " $l$ "

Se demuestra en Geometría, es

$$d_4 = \frac{\sqrt{2}}{2} l = 0,70710678... l$$

Para el caso del dibujo, será:  $d_4 = 0,70710678... \times 34,79 = 24,6 \text{ mm}$

Radio " $d_6$ " de la circunferencia circunscrita a una cara hexagonal de lado " $l$ "

Se demuestra en Geometría, es

$$d_6 = l$$

Radio " $C_4$ " de la esfera tangente a las caras cuadradas de lado " $l$ "

Se obtiene aplicando la fórmula general [2] (ver lám. 33)

$$C_4 = \sqrt{a^2 - (d_4)^2} = \sqrt{\left(\frac{\sqrt{10}}{2} l\right)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{\frac{10}{4} - \frac{2}{4}} \times l = \sqrt{\frac{8}{4}} \times l =$$

$$= \sqrt{2} l = 1,41421356... l$$

Para el caso del dibujo, será:  $C_4 = 1,41421356... \times 34,79 = 49,2 \text{ mm}$

The first part of the book is devoted to the study of the properties of the real numbers. It is shown that the real numbers form a complete ordered field. This means that they satisfy the following properties:

$$1. \text{ Closure: } a + b, ab \in \mathbb{R} \text{ for } a, b \in \mathbb{R}.$$

2. Associativity:  $(a + b) + c = a + (b + c)$  and  $(ab)c = a(bc)$  for  $a, b, c \in \mathbb{R}$ .

3. Commutativity:  $a + b = b + a$  and  $ab = ba$  for  $a, b \in \mathbb{R}$ .

4. Distributivity:  $a(b + c) = ab + ac$  for  $a, b, c \in \mathbb{R}$ .

5. Identity:  $a + 0 = a$  and  $a \cdot 1 = a$  for  $a \in \mathbb{R}$ .

6. Inverse: For every  $a \in \mathbb{R}$ , there exists  $-a$  such that  $a + (-a) = 0$  and  $a \neq 0$  implies there exists  $a^{-1}$  such that  $aa^{-1} = 1$ .

7. Order: For any  $a, b \in \mathbb{R}$ , exactly one of the following holds:  $a < b$ ,  $a = b$ , or  $a > b$ .

8. Completeness: Every non-empty set of real numbers which is bounded above has a least upper bound (supremum).

Radio "C<sub>6</sub>" de la esfera tangente a las caras hexagonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33)

$$C_6 = \sqrt{a^2 - (d_6)^2} = \sqrt{\left(\frac{\sqrt{10}}{2} l\right)^2 - l^2} = \sqrt{\frac{10}{4} - 1} \cdot l = \sqrt{\frac{5}{2} - 1} \cdot l = \sqrt{\frac{3}{2}} \cdot l =$$

$$= \sqrt{\frac{6}{4}} \cdot l = \boxed{\frac{\sqrt{6}}{2} l} = 1, 22 \ 47 \ 44 \ 87 \dots l$$

Para el caso del dibujo, sea

$$C_6 = 1, 22 \ 47 \ 44 \ 87 \dots \cdot 34, 79 = 42, 6 \text{ m.}$$

Ángulo rectilíneo "α<sub>4</sub>" del diedro formado por una cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{\operatorname{tg} \alpha_4} = \frac{2 C_4}{\sqrt{4 (d_4)^2 - l^2}} = \frac{2 \sqrt{2} l}{\sqrt{4 \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{2 \sqrt{2}}{\sqrt{4 \cdot \frac{1}{2} - 1}} =$$

$$= \boxed{2 \sqrt{2}} = 2, 82 \ 84 \ 27 \ 12 \dots$$

$$l, \operatorname{tg} \alpha_4 = 0, 45 \ 15 \ 45 \ 0$$

$$\boxed{\alpha_4 = 70^\circ \ 31' \ 43, 6''}$$

Ángulo rectilíneo "α<sub>6</sub>" del diedro formado por una cara hexagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella

The first part of the document discusses the importance of maintaining accurate records of all transactions. It is essential for the company to have a clear and concise system in place to ensure that all data is properly recorded and stored. This will help in the future when it comes to analyzing the data and making informed decisions.

The second part of the document focuses on the need for transparency and accountability. It is important for the company to be open and honest about its operations and financials. This will help to build trust with the stakeholders and ensure that the company is operating in a responsible and ethical manner.

The third part of the document discusses the importance of communication and collaboration. It is essential for the company to have a strong communication system in place to ensure that all team members are kept up to date on the latest developments. This will help to foster a sense of teamwork and ensure that everyone is working towards the same goals.

The fourth part of the document focuses on the need for innovation and creativity. It is important for the company to be open to new ideas and approaches. This will help to ensure that the company is always at the forefront of its industry and is able to adapt to changing market conditions.

The fifth part of the document discusses the importance of risk management. It is essential for the company to have a clear understanding of the risks it faces and to have a plan in place to mitigate them. This will help to ensure that the company is able to navigate any challenges that may arise.

The sixth part of the document focuses on the need for continuous improvement. It is important for the company to regularly evaluate its performance and make adjustments as needed. This will help to ensure that the company is always striving for excellence and is able to stay ahead of the competition.

The seventh part of the document discusses the importance of customer satisfaction. It is essential for the company to have a strong focus on the needs and wants of its customers. This will help to ensure that the company is able to provide a high-quality product or service that meets the expectations of its customers.

The eighth part of the document focuses on the need for financial stability. It is important for the company to have a clear understanding of its financial situation and to have a plan in place to ensure that it is able to meet its financial obligations. This will help to ensure that the company is able to sustain its operations in the long term.

The ninth part of the document discusses the importance of legal compliance. It is essential for the company to have a clear understanding of the laws and regulations that apply to its industry. This will help to ensure that the company is able to operate in a legal and ethical manner.

The tenth part of the document focuses on the need for a strong corporate culture. It is important for the company to have a clear set of values and beliefs that guide its operations. This will help to ensure that the company is able to attract and retain top talent and is able to create a positive work environment for all employees.

Se obtiene, en función de su tangente, por la fórmula general [6] (ver lám. 33)

$$\boxed{\tan \alpha_6} = \frac{2c_6}{\sqrt{4(d_6)^2 - l^2}} = \frac{2 \times \frac{\sqrt{6}}{2} l}{\sqrt{4l^2 - l^2}} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2} = 1,41421356...$$

$$\tan \frac{1}{2} \alpha_6 = 0,1505150$$

$$\boxed{\alpha_6 = 54^\circ 44' 8,2''}$$

Ángulo rectilíneo " $\varphi_{4-6}$ " del diedro formado por una cara cuadrada y otra exagonal regular

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{4-6}} = \alpha_4 + \alpha_6 = 70^\circ 31' 43,6'' + 54^\circ 44' 8,2'' =$$

$$= \boxed{125^\circ 15' 51,8''}$$

También puede obtenerse directamente, así:

$$\boxed{\tan \varphi_{4-6}} = \tan (\alpha_4 + \alpha_6) = \frac{\tan \alpha_4 + \tan \alpha_6}{1 - \tan \alpha_4 \tan \alpha_6} = \frac{2\sqrt{2} + \sqrt{2}}{1 - 2\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{1-4} =$$

$$= -\frac{3\sqrt{2}}{3} = \boxed{-\sqrt{2}} \quad \text{y haciendo } \alpha_0 = \pi - \varphi_{4-6}, \text{ será:}$$

$$\tan \alpha_0 = -\tan \varphi_{4-6} = -(-\sqrt{2}) = \sqrt{2} \quad \tan \frac{1}{2} \alpha_0 = 0,1505150$$

$\alpha_0 = 54^\circ 44' 8,2''$ , por lo que será:

$$\boxed{\varphi_{4-6}} = 180^\circ - 54^\circ 44' 8,2'' = \boxed{125^\circ 15' 51,8''}$$



10

THE UNIVERSITY OF CHICAGO

1911

RECEIVED OF THE UNIVERSITY OF CHICAGO

THE SUM OF FIVE DOLLARS

PAID TO THE

LIBRARY

FOR THE PURCHASE OF BOOKS

AND MANUSCRIPTS

THE UNIVERSITY OF CHICAGO

LIBRARY

CHICAGO, ILL.

1911

THE UNIVERSITY OF CHICAGO

LIBRARY

CHICAGO, ILL.

Ángulo rectilíneo " $\varphi_{6-6}$ " del diedro formado por dos caras exagonales regulares.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{6-6}} = \alpha_6 + \alpha_6 = 2 \alpha_6 = 2 \times (54^\circ 44' 8.2'') =$$

$$= \boxed{109^\circ 28' 16.4''}$$

También puede obtenerse directamente así:

$$\boxed{\frac{1}{2} \varphi_{6-6}} = \frac{1}{2} 2 \alpha_6 = \frac{2 \frac{1}{2} \alpha_6}{1 - \frac{1}{2} \alpha_6} = \frac{2 \sqrt{2}}{1 - (\sqrt{2})^2} = \frac{2 \sqrt{2}}{-1} = \boxed{-2 \sqrt{2}}$$

y haciendo  $\alpha_0 = \pi - \varphi_{6-6}$ , será  $\frac{1}{2} \alpha_0 = -\frac{1}{2} \varphi_{6-6} = -(-2 \sqrt{2}) = 2 \sqrt{2}$

$$\lg \frac{1}{2} \alpha_0 = \lg (2 \sqrt{2}) = \lg 2, 82 84 27 12 \dots = 0, 45 15 45 0$$

$\alpha_0 = 70^\circ 31' 43,6''$  por lo que será:

$$\boxed{\varphi_{6-6}} = 180^\circ - 70^\circ 31' 43,6'' = \boxed{109^\circ 28' 16,4''}$$

Área lateral "5" del arquimédiano

Se compone de la suma de 6 caras cuadradas y 8 exagonales regulares, todas de lado "l". La apotema de la cara exagonal, será:



THE UNIVERSITY OF CHICAGO  
CHICAGO, ILL.

TO THE HONORABLE THE SENATE

OF THE UNIVERSITY OF CHICAGO

FOR THE YEAR 1912

IN RESPONSE TO A RESOLUTION

PASSED BY THE SENATE AT ITS MEETING

Held at Chicago, Ill., on the 15th day of May, 1912

AND IN ACCORDANCE WITH THE

BY-LAWS OF THE UNIVERSITY

OF CHICAGO, ILL.



WILLIAM D. HENNING, President

W. D. HENNING, President  
of the University of Chicago  
Chicago, Ill.

$$\text{apotema} = \sqrt{(d_6)^2 - \left(\frac{l}{2}\right)^2} = \sqrt{l^2 - \frac{l^2}{4}} = \sqrt{1 - \frac{1}{4}} \cdot l = \sqrt{\frac{3}{4}} l = \frac{\sqrt{3}}{2} l$$

por lo que el área total, será:

$$[S] = 6 \times l^2 + 8 \times \frac{6}{2} \times \frac{\sqrt{3}}{2} l^2 = (6 + 12\sqrt{3}) l^2 = \boxed{6 \times (1 + 2\sqrt{3}) l^2} =$$

$$= 26, 78 \ 46 \ 09 \ 73$$

### Volumen "V" del arquimédiano

Se compone de la suma de 6 pirámides regulares, de base cuadrada y altura " $C_4$ ", y de 8 pirámides hexagonales regulares y altura " $C_6$ ". (lado " $l$ "). Su volumen será pues:

$$[V] = 6 l^2 \times \frac{C_4}{3} + 12 \sqrt{3} l^2 \times \frac{C_6}{3} = 2 l^2 \times \sqrt{2} l + 4 \sqrt{3} \times \frac{\sqrt{6}}{2} l =$$

$$= (2\sqrt{2} + 2\sqrt{18}) l^3 = (2\sqrt{2} + 6\sqrt{2}) l^3 = \boxed{8\sqrt{2} l^3} = 11, 31 \ 37 \ 08 \ 50... l^3$$

### FIGURA CORPÓREA

Se obtiene por acoplamiento de 6 cuadrados de lado  $l = 34,8 \text{ mm}$  y 8 hexágonos regulares de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren 2 hexágonos y un cuadrado.

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En el cuadro sinóptico que damos a continuación, se resumen los resultados analíticos obtenidos anteriormente.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\frac{\sqrt{10}}{2} \ell$	1.58 11 39... $\ell$
$b$	$\frac{3}{2} \ell$	1.50 00 00... $\ell$
$c_4$	$\sqrt{2} \ell$	1.41 42 14... $\ell$
$c_6$	$\frac{\sqrt{6}}{2} \ell$	1.22 47 45... $\ell$
$d_4$	$\frac{\sqrt{2}}{2} \ell$	0.70 71 07... $\ell$
$d_6$	$1 \ell$	1.00 00 00... $\ell$
$m$	$\frac{3\sqrt{10}}{10} \ell$	0.94 86 83... $\ell$
$\alpha_4$	$\text{tg } \alpha_4 = 2\sqrt{2}$	$\text{tg } \alpha_4 = 2.82 84 27...$ $\alpha_4 = 70^\circ 31' 43.6''$
$\alpha_6$	$\text{tg } \alpha_6 = \sqrt{2}$	$\text{tg } \alpha_6 = 1.41 42 14...$ $\alpha_6 = 54^\circ 44' 8.2''$
$\varphi_{4-6}$	$\text{tg } \varphi_{4-6} = -\sqrt{2}$	$\text{tg } \varphi_{4-6} = -1.41 42 14$ $\varphi_{4-6} = 125^\circ 15' 51.8''$
$\varphi_{6-6}$	$\text{tg } \varphi_{6-6} = -2\sqrt{2}$	$\text{tg } \varphi_{6-6} = -2.82 84 27$ $\varphi_{6-6} = 109^\circ 28' 16.4''$
$S$	$6(1+2\sqrt{3}) \ell^2$	26, 78 46 10... $\ell^2$
$V$	$8\sqrt{2} \ell^3$	11, 31 37 09... $\ell^3$

The following table shows the results of the experiment conducted on the 10th of May 1900. The results are given in the following table.

Table showing the results of the experiment.

Time	Temperature	Pressure
10.00	10.0	10.0
10.10	10.1	10.1
10.20	10.2	10.2
10.30	10.3	10.3
10.40	10.4	10.4
10.50	10.5	10.5
11.00	10.6	10.6
11.10	10.7	10.7
11.20	10.8	10.8
11.30	10.9	10.9
11.40	11.0	11.0
11.50	11.1	11.1
12.00	11.2	11.2
12.10	11.3	11.3
12.20	11.4	11.4
12.30	11.5	11.5
12.40	11.6	11.6
12.50	11.7	11.7
13.00	11.8	11.8
13.10	11.9	11.9
13.20	12.0	12.0
13.30	12.1	12.1
13.40	12.2	12.2
13.50	12.3	12.3
14.00	12.4	12.4
14.10	12.5	12.5
14.20	12.6	12.6
14.30	12.7	12.7
14.40	12.8	12.8
14.50	12.9	12.9
15.00	13.0	13.0

PROCESO GRÁFICO-ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder, en la lámina 42, a la representación gráfica del Arquimediano X.

Para su trazado nos valdremos de estas calculadas por las fórmulas anteriores, y de procesos gráficos.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_x = \text{Dato del ejercicio} = 34,8 \text{ mm}$$

$$a = 1,58 \ 11 \ 39 \dots \times 34,79 = 55,0 \text{ mm}$$

$$b = 1,5 \dots \times 34,79 = 52,2 \text{ mm}$$

$$C_4 = 1,41 \ 42 \ 14 \dots \times 34,79 = 49,2 \text{ mm}$$

$$C_6 = 1,22 \ 47 \ 45 \dots \times 34,79 = 42,6 \text{ mm}$$

$$d_4 = 0,70 \ 71 \ 07 \dots \times 34,79 = 24,6 \text{ mm}$$

$$d_6 = 1 \dots \times 34,79 = 34,8 \text{ mm}$$

El orden de operaciones del trazado gráfico (lámin. 42), es el siguiente:

1° Situar el centro O, de coordenadas O (72, 72, 85) mm.

2° Dibujar en I, II y III las proyecciones de la esfera circunscrita, de radio  $a = 55,0 \text{ mm}$ .

3° Representar en I, II y III las caras cuadradas superior 1 al 4 e inferior 21 al 24, supuesto el poliedro colocado con dichas caras paralelas a II, y un lado per-

My dear \_\_\_\_\_

I am very glad to hear from you and hope you are well. I am feeling better now, but still a bit weak. I am going to the office tomorrow and will try to get some work done. I will write again soon.

Yours truly,

\_\_\_\_\_

I am very glad to hear from you and hope you are well. I am feeling better now, but still a bit weak. I am going to the office tomorrow and will try to get some work done. I will write again soon.

Yours truly,

\_\_\_\_\_

pendicular a I (utilícese la cota " $c_4$ " en I y III, y la " $d_4$ " en II).

4° Representar en II las aristas 9-10, 11-12, 13-14 y 15-16, que son paralelas a II y de longitud " $l$ "; los extremos de dichas aristas están en II sobre el contorno aparente de la esfera circunscrita de radio " $a$ "; determinar seguidamente las proyecciones de ellas en I y III.

5° Completar el contorno aparente de la proyección II y trazar las aristas intermedias 1-6, 2-7, 3-8, 4-5, prolongaciones de las diagonales del cuadrado 1 al 4, que cortarán a las aristas 10-11, 12-13, 14-15, 16-9 en sus respectivos puntos medios. Con estos trazados queda completada la proyección en II

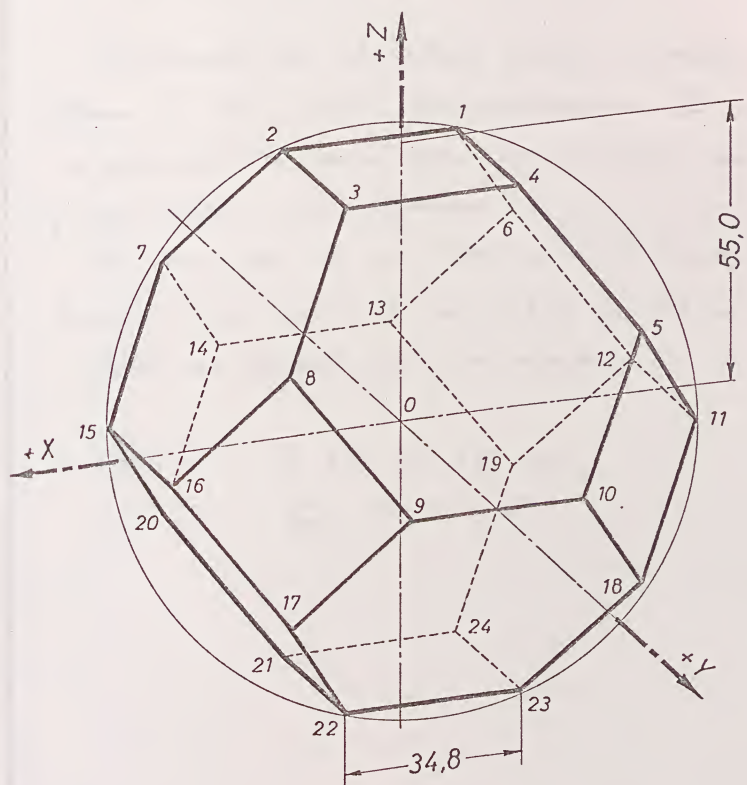
6° Completar las proyecciones I y III del arquimedianos (son iguales), por correspondencia con los vértices de II: Compruébese que los vértices 5 al 8 y 17 al 20 distan de O la magnitud " $\frac{c_4}{2}$ "



The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The second part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The third part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The fourth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The fifth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter.

The sixth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The seventh part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The eighth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The ninth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The tenth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter.

The eleventh part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The twelfth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The thirteenth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The fourteenth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter. The fifteenth part of the paper is devoted to a detailed study of the problem. It is shown that the problem is of great importance in the theory of the structure of matter.



Arquimediano X



Geometrische Optik

ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el arquimediano XI, en el que en cada vértice concurren un cuadrado, un hexágono y un octógono, todos regulares.

La longitud de su lado es de 23,7 mm, y las coordenadas de su centro O, son O(72, 72, 85) mm.

Diblar en formato A3V y a escala 1:1

DATOS

O (72, 72, 85) mm

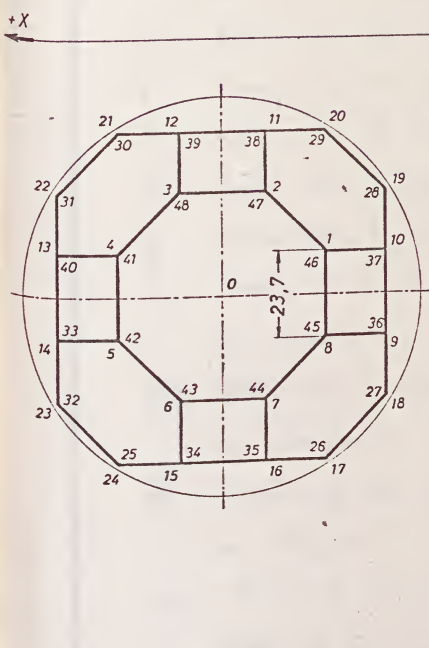
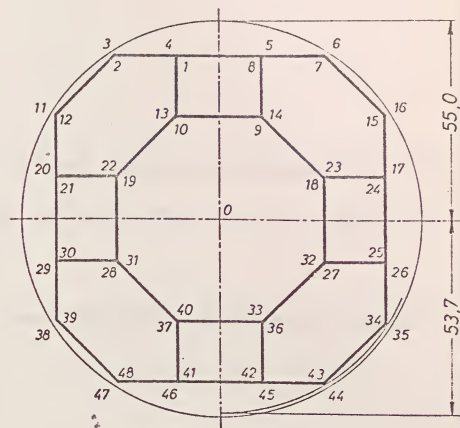
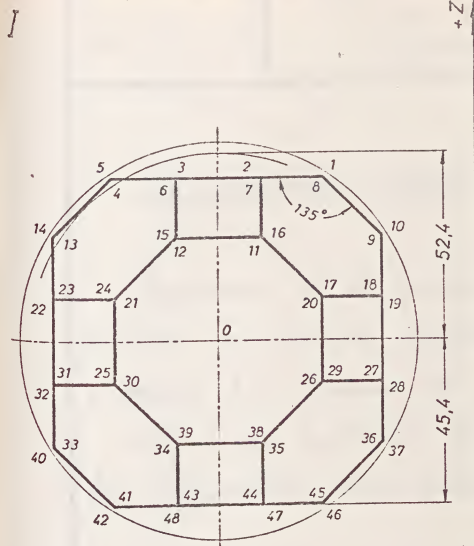
 $l_{XI} = 23,7 \text{ mm}$

Chicago

My dear Mr. [Name] I have just received your letter of the 10th inst. and am glad to hear that you are well. I am also well and hope this letter finds you the same. I have been thinking of you very much lately and wondering how you are getting on. I hope you are still in the same good health and enjoying your work. I have been very busy lately but I will try to write you more often. I am sure you will understand. I am, dear Mr. [Name], very truly yours,

[Signature]





#### ARQUIMEDIANO XI

Número de caras cuadradas.....  $C_4 = 12$   
 Número de caras exagonales.....  $C_6 = 8$   
 Número de caras octogonales.....  $C_8 = 6$   
 Número de vértices.....  $V = 48$   
 Número de aristas.....  $A = 72$   
 Número de caras de un ángulo sólido:  $1P_4 + 1P_6 + 1P_8$

#### ENUNCIADO

Representar por el método gráfico-analítico, en los planos I, II y III, el Arquimediano XI, en el que en cada cara concurren un cuadrado, un exágono y un octógono, todos regulares.

La longitud de su lado es de 23,7 milímetros y las coordenadas de su centro O, son: O (72, 72, 85) mm.

Dibujar en formato A3v y a escala 1:1.

Propuesta	De entrega	Entregada	Calificación	(firma)	Escuela Curso
Fecha:					
Alumno:					
Escala	Arquimediano XI				Lámina 43
1:1					Curso 19 - 19



1. The first diagram is a circle with a cross inside, divided into four quadrants. The cross is formed by two perpendicular lines intersecting at the center. The quadrants are labeled with letters: 'A' in the top-left, 'B' in the top-right, 'C' in the bottom-left, and 'D' in the bottom-right.

2. The second diagram is a circle with a cross inside, divided into four quadrants. The cross is formed by two perpendicular lines intersecting at the center. The quadrants are labeled with letters: 'A' in the top-left, 'B' in the top-right, 'C' in the bottom-left, and 'D' in the bottom-right.

CONSIDERACIONES PREVIAS

Sequircmos en el estudio de este arquimediano, las directrices y fórmulas generales planteadas en el "Arquimediano I", lámina 33.

En el caso particular que nos ocupa, determinaremos las magnitudes siguientes:

$l$  = Arista del Arquimediano XI (dato del ejercicio).

$a$  = Radio de la esfera circunscrita.

$b$  = Radio de la esfera tangente a las aristas.

$c_4$  = Radio de la esfera tangente a las caras cuadradas.

$c_6$  = Radio de la esfera tangente a las caras hexagonales.

$c_8$  = Radio de la esfera tangente a las caras octogonales.

$d_4$  = Radio de la circunferencia circunscrita a una cara cuadrada.

$d_6$  = Radio de la circunferencia circunscrita a una cara hexagonal.

$d_8$  = Radio de la circunferencia circunscrita a una cara octogonal.

$m$  = Radio de la circunferencia circunscrita al polígono obtenido al unir los extremos de las aristas de un ángulo sólido.

$\alpha_4$  = Ángulo rectilíneo del diedro formado por una



cara cuadrada, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\alpha_6$  = Ángulo rectilíneo del diedro formado por una cara hexagonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\alpha_8$  = Ángulo rectilíneo del diedro formado por una cara octogonal, con el plano diametral del arquimedianos que pasa por una arista de aquella.

$\varphi_{4-6}$  = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra hexagonal.

$\varphi_{4-8}$  = Ángulo rectilíneo del diedro formado por una cara cuadrada y otra octogonal.

$\varphi_{6-8}$  = Ángulo rectilíneo del diedro formado por una cara hexagonal y otra octogonal.

$S$  = Superficie

$V$  = Volumen

### PROCESO GRÁFICO-ANALÍTICO

El estudio realizado de este arquimedianos, nos indica que se compone de 12 caras cuadradas, 8 caras hexagonales y 6 caras octogonales; 48 vértices y 72 aristas.

En cada vértice concurren un cuadrado, un hexágono y un octógono, todos regulares.

Aquí pues, tendremos que:





ARQUIMEDIANO XI ( $1P_4 + 1P_6 + 1P_8$ );  $C_4 = 12$ ;  $C_6 = 8$ ;  $C_8 = 6$ ;  $V = 48$ ;  $A = 72$

Cálculo de sus magnitudes

Arista "l" del arquimedianos

Dato del ejercicio

Radio "m" de la circunferencia circunscrita al polígono  
obtenido al unir los extremos de las tres aristas que  
convergen en un ángulo sólido.

Este polígono es un triángulo

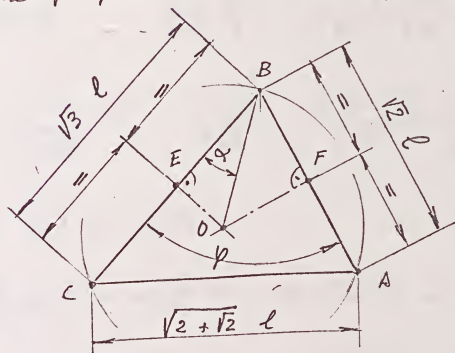


Figura 1

A-B-C (fig. 1) escaleno, cuyos lados  $\overline{AB}$ ,  $\overline{BC}$  y  $\overline{CA}$  son respectivamente las diagonales que unen tres vértices consecutivos en un cuadrado, un hexágono y un octógono regulares, todos de lado "l".

Se demuestra en Geometría que estas diagonales son:

a) En el cuadrado

$$AB = \sqrt{2} \quad l$$

[1]

b) En el hexágono

$$BC = \sqrt{3} \quad l$$

[2]

Subject: Mathematics

Chapter: Geometry

Topic: Area and Perimeter

Q.1. Find the area of a rectangle with length 10 cm and breadth 5 cm.

Q.2. Find the perimeter of a square with side 4 cm.



c) En el octógono

$$AC = \sqrt{2 + \sqrt{2}} \cdot l \quad [3]$$

De la figura 1, se deduce:

$$\overline{BO} = m = \frac{\overline{BE}}{\cos \alpha} = \frac{\overline{BF}}{\cos (\varphi - \alpha)} \quad [4]$$

de donde

$$\overline{BE} \times \cos (\varphi - \alpha) = \overline{BF} \times \cos \alpha \quad "$$

$$\frac{\sqrt{3}}{2} l \cos (\varphi - \alpha) = \frac{\sqrt{2}}{2} l \cos \alpha \quad " \quad \sqrt{3} \cos (\varphi - \alpha) = \sqrt{2} \cos \alpha$$

$$\cos (\varphi - \alpha) = \frac{\sqrt{2}}{3} \cdot \cos \alpha \quad [5]$$

por otra parte tenemos:

$$\overline{AC}^2 = \overline{BC}^2 + \overline{BA}^2 - 2 \times \overline{BC} \times \overline{BA} \times \cos \varphi \quad " \quad \overline{BC}^2 + \overline{BA}^2 - \overline{AC}^2 = 2 \times \overline{BC} \times \overline{BA} \times \cos \varphi$$

$$\text{de donde} \quad \boxed{\cos \varphi} = \frac{\overline{BC}^2 + \overline{BA}^2 - \overline{AC}^2}{2 \times \overline{BC} \times \overline{BA}} = \frac{(\sqrt{3} l)^2 + (\sqrt{2} l)^2 - (\sqrt{2 + \sqrt{2}} l)^2}{2 \sqrt{3} l \times \sqrt{2} l} =$$

$$= \frac{3 + 2 - (2 + \sqrt{2})}{2 \sqrt{6}} = \frac{3 - \sqrt{2}}{2 \sqrt{6}} = \frac{3 \sqrt{6} - \sqrt{12}}{12} = \frac{3 \sqrt{6} - 2 \sqrt{3}}{12} \quad [6]$$

0,3236

De [5] se deduce:

$$\frac{\sqrt{2}}{3} \cos \alpha = \cos (\varphi - \alpha) = \cos \varphi \cos \alpha + \sin \varphi \sin \alpha \quad "$$

$$\frac{\sqrt{2}}{3} \cos \alpha = \cos \varphi \cos \alpha + \sqrt{1 - \cos^2 \varphi} \times \sqrt{1 - \cos^2 \alpha} \quad "$$

$$\frac{\sqrt{2}}{3} \cos \alpha - \cos \varphi \cos \alpha = \sqrt{(1 - \cos^2 \varphi)(1 - \cos^2 \alpha)} \quad "$$

$$\left[ \left( \frac{\sqrt{2}}{3} - \cos \varphi \right) \cos \alpha \right]^2 = (1 - \cos^2 \varphi)(1 - \cos^2 \alpha) \quad "$$





$$\left(\sqrt{\frac{2}{3}} - \cos \varphi\right)^2 \cos^2 \alpha = (1 - \cos^2 \varphi) - (1 - \cos^2 \varphi) \cos^2 \alpha \quad "$$

$$\left(\sqrt{\frac{2}{3}} - \cos \varphi\right)^2 \cos^2 \alpha + (1 - \cos^2 \varphi) \cos^2 \alpha = 1 - \cos^2 \varphi \quad "$$

$$\left[\left(\sqrt{\frac{2}{3}} - \cos \varphi\right)^2 + (1 - \cos^2 \varphi)\right] \cos^2 \alpha = 1 - \cos^2 \varphi \quad "$$

$$\cos^2 \alpha = \frac{1 - \cos^2 \varphi}{\left(\sqrt{\frac{2}{3}} - \cos \varphi\right)^2 + (1 - \cos^2 \varphi)} = \frac{1 - \cos^2 \varphi}{\frac{2}{3} + \cos^2 \varphi - 2\sqrt{\frac{2}{3}} \cos \varphi + 1 - \cos^2 \varphi} =$$

$$= \frac{1 - \cos^2 \varphi}{\frac{2}{3} + 1 - 2\sqrt{\frac{2}{3}} \cos \varphi} = \frac{1 - \cos^2 \varphi}{\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi} \quad \text{y finalmente}$$

$$\cos \alpha = \sqrt{\frac{1 - \cos^2 \varphi}{\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi}} \quad [7]$$

solo que sustituido en [4], me da

$$m = \frac{BF}{\cos \alpha} = \frac{\sqrt{3}}{2} l : \sqrt{\frac{1 - \cos^2 \varphi}{\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi}} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 : \frac{1 - \cos^2 \varphi}{\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi}} \times l =$$

$$= \sqrt{\frac{\frac{3}{4} : \frac{1 - \cos^2 \varphi}{\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi}}{1 - \cos^2 \varphi}} \times l = \sqrt{\frac{3 \times \left(\frac{5}{3} - 2\sqrt{\frac{2}{3}} \cos \varphi\right)}{4(1 - \cos^2 \varphi)}} \times l =$$

$$= \sqrt{\frac{5 - 6\sqrt{\frac{2}{3}} \cos \varphi}{4(1 - \cos^2 \varphi)}} \times l \quad \text{y teniendo en cuenta [6], el [8]}$$

numerador:  $\underline{5 - 6\sqrt{\frac{2}{3}} \cos \varphi} = 5 - 6\sqrt{\frac{2}{3}} \times \frac{3\sqrt{6} - 2\sqrt{3}}{12} =$

Let  $f(x) = \frac{1}{x^2} - \frac{1}{x}$ . Then  $f(1) = \frac{1}{1^2} - \frac{1}{1} = 1 - 1 = 0$ .

Now, let's find the value of the function  $f(x)$  at  $x = 2$ .

Let  $f(x) = \frac{1}{x^2} - \frac{1}{x}$ . Then  $f(2) = \frac{1}{2^2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$ .

$$f(x) = \frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

$$f(x) = \frac{1 - x}{x^2} = \frac{1}{x^2} - \frac{1}{x}$$

$$\frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

Let's find the value of the function  $f(x)$  at  $x = 3$ .

$$\frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

$$\frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

$$\frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

$$\frac{1}{x^2} - \frac{1}{x} = \frac{1 - x}{x^2}$$

$$= 5 - 6 \cdot \frac{\sqrt{2} (3\sqrt{6} - 2\sqrt{3})}{12\sqrt{3}} = 5 - \frac{3\sqrt{12} - 2\sqrt{6}}{2\sqrt{3}} = 5 - \frac{3\sqrt{36} - 2\sqrt{18}}{6} = 5 - \frac{18 - 6\sqrt{2}}{6} =$$

$$= 5 - (3 - \sqrt{2}) = \underline{2 + \sqrt{2}} \quad [9]$$

y también el denominador

$$\begin{aligned} 4(1 - \cos^2 \varphi) &= 4 \left[ 1 - \left( \frac{3\sqrt{6} - 2\sqrt{3}}{12} \right)^2 \right] = 4 \cdot \left( 1 - \frac{54 + 12 - 12\sqrt{18}}{144} \right) = 4 \left( 1 - \frac{66 - 36\sqrt{2}}{144} \right) = \\ &= 4 \cdot \frac{144 - 66 + 36\sqrt{2}}{144} = 4 \cdot \frac{78 + 36\sqrt{2}}{144} = 4 \cdot \frac{13 + 6\sqrt{2}}{24} = \underline{\underline{\frac{13 + 6\sqrt{2}}{6}}} \quad [10] \end{aligned}$$

sustituyendo los valores [9] y [10] en [8], tenemos finalmente

$$\begin{aligned} [m] &= \sqrt{\frac{2 + \sqrt{2}}{\frac{13 + 6\sqrt{2}}{6}}} \cdot \ell = \sqrt{\frac{6(2 + \sqrt{2})}{13 + 6\sqrt{2}}} \cdot \ell = \sqrt{\frac{6(2 + \sqrt{2})(13 - 6\sqrt{2})}{13^2 - 72}} \cdot \ell \\ &= \sqrt{\frac{6(26 + 13\sqrt{2} - 12\sqrt{2} - 12)}{97}} \cdot \ell = \boxed{\sqrt{\frac{6(14 + \sqrt{2})}{97}}} \cdot \ell = 0,97645097... \cdot \ell \end{aligned}$$

Para el caso del dibujo, será:  $m = 0,97645097... \cdot x$

Radio "a" de la esfera circunscrita

Se obtiene aplicando la fórmula general [7] (ver lám. 33)

$$\begin{aligned} [a] &= \frac{\ell^2}{2\sqrt{\ell^2 - m^2}} = \frac{\ell^2}{2\sqrt{\ell^2 - \left( \sqrt{\frac{6(14 + \sqrt{2})}{97}} \ell \right)^2}} = \frac{1}{2\sqrt{1 - \frac{6(14 + \sqrt{2})}{97}}} \cdot \ell = \\ &= \frac{1}{2\sqrt{\frac{97 - 84 - 6\sqrt{2}}{97}}} \cdot \ell = \frac{1}{2} \sqrt{\frac{97}{13 - 6\sqrt{2}}} \cdot \ell = \frac{1}{2} \sqrt{\frac{97(13 + 6\sqrt{2})}{13^2 - 72}} \cdot \ell = \end{aligned}$$

Date	Particulars	Amount
1890	To Balance	100.00
1891	By Cash	50.00
1892	To Cash	25.00
1893	By Cash	75.00
1894	To Cash	100.00
1895	By Cash	150.00
1896	To Cash	200.00
1897	By Cash	250.00
1898	To Cash	300.00
1899	By Cash	350.00
1900	To Cash	400.00
1901	By Cash	450.00
1902	To Cash	500.00
1903	By Cash	550.00
1904	To Cash	600.00

$$= \frac{1}{2} \sqrt{\frac{97(13+6\sqrt{2})}{97}} \cdot l = \frac{1}{2} \sqrt{13+6\sqrt{2}} \cdot l = \boxed{\frac{\sqrt{13+6\sqrt{2}}}{2} l} = 2,31761091... l$$

Para el caso del dibujo, será:  $a = 55 \text{ mm}$   $l = 23,73 \text{ mm}$ .

Radio "b" de la esfera tangente a las aristas

Se obtiene aplicando la fórmula general [3] (ver lám. 33)

$$\boxed{b} = \sqrt{a^2 - \frac{l^2}{4}} = \sqrt{\left(\frac{\sqrt{13+6\sqrt{2}}}{2} l\right)^2 - \frac{l^2}{4}} = \sqrt{\frac{13+6\sqrt{2}}{4} - \frac{1}{4}} \times l =$$

$$= \sqrt{\frac{12+6\sqrt{2}}{4}} l = \boxed{\frac{\sqrt{6+3\sqrt{2}}}{2} \times l} = 2,26303344... l$$

Para el caso del dibujo, será:  $b = 2,26303344... \times 23,73 = 53,7 \text{ mm}$

Radio "d<sub>4</sub>" de la circunferencia circunscrita a una cara cuadrada de lado "l".

Se demuestra en geometría, es

$$\boxed{d_4 = \frac{\sqrt{2}}{2} l} = 0,70710678... l$$

Para el caso del dibujo, será:  $d_4 = 0,70710678... \times 23,73 = 16,8 \text{ mm}$ .

Radio "d<sub>6</sub>" de la circunferencia circunscrita a una cara hexagonal de lado "l"

Se demuestra en geometría, es:

$$\boxed{d_6 = l}$$



1-2000 1-2000 1-2000 1-2000 1-2000

1-2000 1-2000 1-2000 1-2000 1-2000

1-2000 1-2000 1-2000 1-2000 1-2000

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1-2000 1-2000 1-2000 1-2000 1-2000

Radio " $d_8$ " de la circunferencia circunscrita a una cara octogonal de lado " $l$ "

Se demuestra en Geometría, es

$$d_8 = \sqrt{\frac{2 + \sqrt{2}}{2}} \times l = 1,30656296...l$$

Para el caso del dibujo, será:  $d_8 = 1,30656296... \times 23,73 = 30,99 \text{ mm}$

Radio " $C_4$ " de la esfera tangente a las caras cuadradas de lado " $l$ "

Aplicando la fórmula general [2] (ver lám. 33)

$$C_4 = \sqrt{a^2 - (d_4)^2} = \sqrt{\left(\frac{\sqrt{13+6\sqrt{2}}}{2} l\right)^2 - \left(\frac{\sqrt{2}}{2} l\right)^2} = \sqrt{\frac{13+6\sqrt{2}}{4} - \frac{2}{4}} \times l =$$

$$= \sqrt{\frac{11+6\sqrt{2}}{4}} l = \frac{\sqrt{11+6\sqrt{2}}}{2} l = \frac{\frac{\sqrt{18}}{2} + \frac{\sqrt{4}}{2}}{2} l = \frac{3+\sqrt{2}}{2} l = 2,20710678...l$$

Para el caso del dibujo, será:  $C_4 = 2,20710678... \times 23,73 = 52,4 \text{ mm}$

Radio " $C_6$ " de la esfera tangente a las caras hexagonales de lado " $l$ "

Aplicando la fórmula general [2] (ver lám. 33)

$$C_6 = \sqrt{a^2 - (d_6)^2} = \sqrt{\left(\frac{\sqrt{13+6\sqrt{2}}}{2} l\right)^2 - l^2} = \sqrt{\frac{13+6\sqrt{2}}{4} - 1} \times l =$$

$$= \sqrt{\frac{9+6\sqrt{2}}{4}} l = \frac{\sqrt{9+6\sqrt{2}}}{2} l = \frac{\frac{\sqrt{12}}{2} + \frac{\sqrt{6}}{2}}{2} l = \frac{\sqrt{6} + \sqrt{3}}{2} l =$$

Let  $x$  and  $y$  be any two numbers. Then we have

$$x + y = \text{sum}$$

$$x - y = \text{difference}$$

Adding these two equations, we get

$$2x = \text{sum} + \text{difference}$$

Dividing both sides by 2, we get

$$x = \frac{\text{sum} + \text{difference}}{2}$$

$$y = \frac{\text{sum} - \text{difference}}{2}$$

Thus, we have found the values of  $x$  and  $y$ .

Example: If the sum of two numbers is 10 and their difference is 2, find the numbers.

Solution: Let the two numbers be  $x$  and  $y$ .

$$x + y = 10$$

$$x - y = 2$$

$$= 2,09\ 07\ 70\ 25... l$$

Para el caso del dibujo, será:  $C_6 = 2,09\ 07\ 70\ 25... \times 23,73 = 49,6\ mm$

Radio "C<sub>6</sub>" de la esfera tangente a las caras octogonales de lado "l"

Aplicando la fórmula general [2] (ver lám. 33)

$$C_8 = \sqrt{a^2 - (d_8)^2} = \sqrt{\left(\frac{\sqrt{13+6\sqrt{2}}}{2} l\right)^2 - \left(\frac{\sqrt{2+\sqrt{2}}}{2} l\right)^2} =$$

$$= \sqrt{\frac{13+6\sqrt{2}}{4} - \frac{2+\sqrt{2}}{2}} \cdot l = \sqrt{\frac{13+6\sqrt{2}-4-2\sqrt{2}}{4}} \cdot l = \sqrt{\frac{9+4\sqrt{2}}{4}} \cdot l =$$

$$= \frac{\sqrt{9+4\sqrt{2}}}{2} \cdot l = 1,91\ 42\ 13\ 56... l = \frac{1}{2} \left( \sqrt{\frac{16}{2}} + \sqrt{\frac{2}{2}} \right) l = \frac{2\sqrt{2}+1}{2} l$$

Para el caso del dibujo, será:  $C_8 = 1,91\ 42\ 13\ 56... \times 23,73 = 45,4\ mm$

Ángulo rectilíneo "α<sub>4</sub>" del diedro formado por una cara cuadrada, con el plano diametral del arquimediano que pasa por una arista de aquella.

Se obtiene, en función de su tangente, por la fórmula general [5]. (ver lám. 33)

$$\lg \alpha_4 = \frac{2 C_4}{\sqrt{4 (d_4)^2 - l^2}} = \frac{2 \times \frac{3+\sqrt{2}}{2} l}{\sqrt{4 \left(\frac{\sqrt{2}}{2} l\right)^2 - l^2}} = \frac{3+\sqrt{2}}{\sqrt{4 \times \frac{1}{2} - 1}} = 3 + \sqrt{2} =$$

$$= 4,41\ 42\ 13\ 56...$$

$$\lg \alpha_4 = 0,64\ 48\ 53\ 3$$

$$\alpha_4 = 77^\circ\ 14'\ 8,2''$$

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Ángulo rectilíneo " $\alpha_6$ " del diedro formado por una cara hexagonal, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33).

$$\boxed{\frac{1}{\tan} \alpha_6} = \frac{2 C_6}{\sqrt{4 (d_6)^2 - l^2}} = \frac{2 \times \frac{\sqrt{6} + \sqrt{3}}{2} l}{\sqrt{4 l^2 - l^2}} = \frac{\sqrt{6} + \sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18} + 3}{3} =$$

$$= \frac{3\sqrt{2} + 3}{3} = \boxed{1 + \sqrt{2}} = 2, 41 42 13 56 \dots$$

$$\frac{1}{\tan} \alpha_6 = 0, 38 27 756$$

$$\boxed{\alpha_6 = 67^\circ 30' 0,0''}$$

Ángulo rectilíneo " $\alpha_8$ " del diedro formado por una cara octogonal, con el plano diametral del arquimedianos que pasa por una arista de aquélla.

Se obtiene, en función de su tangente, por la fórmula general [5] (ver lám. 33)

$$\boxed{\frac{1}{\tan} \alpha_8} = \frac{2 C_8}{\sqrt{4 (d_8)^2 - l^2}} = \frac{2 \times \frac{\sqrt{9+4\sqrt{2}}}{2} \times l}{\sqrt{4 \left( \frac{\sqrt{2+\sqrt{2}}}{2} l \right)^2 - l^2}} = \frac{\sqrt{9+4\sqrt{2}}}{\sqrt{4 \times \frac{2+\sqrt{2}}{2} - 1}} =$$

$$= \frac{\sqrt{9+4\sqrt{2}}}{\sqrt{4+2\sqrt{2}-1}} = \frac{\sqrt{9+4\sqrt{2}}}{\sqrt{3+2\sqrt{2}}} = \sqrt{\frac{9+4\sqrt{2}}{3+2\sqrt{2}}} = \sqrt{\frac{(9+4\sqrt{2})(3-2\sqrt{2})}{9-8}} =$$

$$= \sqrt{27 + 12\sqrt{2} - 18\sqrt{2} - 16} = \boxed{\sqrt{11 - 6\sqrt{2}}} = \sqrt{\frac{18}{2}} - \sqrt{\frac{4}{2}} = \boxed{3 - \sqrt{2}} = 1,58 57 86 44 \dots$$

The first part of the question is to find the value of  $x$  and  $y$  such that the given matrix is a magic square. A magic square is a square grid of numbers, usually positive integers, such that the sums of the numbers in each row, each column, and both main diagonals are equal.

$$\begin{aligned}
 & \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \quad \text{--- (1)} \\
 & \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \quad \text{--- (2)}
 \end{aligned}$$

Adding (1) and (2):

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{x} - \frac{1}{y} = \frac{1}{2} + \frac{1}{3}$$

$$\frac{2}{x} = \frac{5}{6} \implies x = \frac{12}{5}$$

Substituting  $x = \frac{12}{5}$  in (1):

$$\frac{1}{\frac{12}{5}} + \frac{1}{y} = \frac{1}{2} \implies \frac{5}{12} + \frac{1}{y} = \frac{1}{2}$$

$$\frac{1}{y} = \frac{1}{2} - \frac{5}{12} = \frac{6}{12} - \frac{5}{12} = \frac{1}{12} \implies y = 12$$

The second part of the question is to find the value of  $x$  and  $y$  such that the given matrix is a magic square. A magic square is a square grid of numbers, usually positive integers, such that the sums of the numbers in each row, each column, and both main diagonals are equal.

$$\begin{aligned}
 & \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \quad \text{--- (1)} \\
 & \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \quad \text{--- (2)}
 \end{aligned}$$

Adding (1) and (2):

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{x} - \frac{1}{y} = \frac{1}{2} + \frac{1}{3}$$

$$\frac{2}{x} = \frac{5}{6} \implies x = \frac{12}{5}$$

Substituting  $x = \frac{12}{5}$  in (1):

$$\frac{1}{\frac{12}{5}} + \frac{1}{y} = \frac{1}{2} \implies \frac{5}{12} + \frac{1}{y} = \frac{1}{2}$$

$$\frac{1}{y} = \frac{1}{2} - \frac{5}{12} = \frac{6}{12} - \frac{5}{12} = \frac{1}{12} \implies y = 12$$

$$\lg \operatorname{tg} \alpha_8 = 0,20\,02\,44\,7$$

$$\alpha_8 = 57^\circ 45' 51,8''$$

Ángulo rectilíneo " $\varphi_{4-6}$ " del diedro formado por una cara cuadrada y otra hexagonal regular.

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{4-6}} = \alpha_4 + \alpha_6 = 77^\circ 14' 8,2'' + 67^\circ 30' 0,0'' =$$

$$= \boxed{144^\circ 44' 8,2''}$$

También puede obtenerse directamente, así:

$$\operatorname{tg} \varphi_{4-6} = \operatorname{tg} (\alpha_4 + \alpha_6) = \frac{\operatorname{tg} \alpha_4 + \operatorname{tg} \alpha_6}{1 - \operatorname{tg} \alpha_4 \operatorname{tg} \alpha_6} = \frac{(3+\sqrt{2}) + (1+\sqrt{2})}{1 - (3+\sqrt{2})(1+\sqrt{2})} =$$

$$= \frac{4 + 2\sqrt{2}}{1 - (3 + \sqrt{2} + 3\sqrt{2} + 2)} = \frac{4 + 2\sqrt{2}}{1 - 5 - 4\sqrt{2}} = - \frac{4 + 2\sqrt{2}}{4 + 4\sqrt{2}} = - \frac{2 + \sqrt{2}}{2 + 2\sqrt{2}} =$$

$$= - \frac{(2 + \sqrt{2})(2\sqrt{2} - 2)}{8 - 4} = - \frac{4\sqrt{2} + 4 - 4 - 2\sqrt{2}}{4} = - \frac{2\sqrt{2}}{4} = - \frac{\sqrt{2}}{2}$$

y haciendo  $\alpha_0 = \pi - \varphi_{4-6}$ , sea:  $\operatorname{tg} \alpha_0 = - \operatorname{tg} \varphi_{4-6} =$

$$= - \left( - \frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} = 0,70\,71\,06\,78 \quad \lg \operatorname{tg} \alpha_0 = 7,84\,94\,85\,0$$

$\alpha_0 = 35^\circ 15' 51,8''$ , por lo que será:

$$\varphi_{4-6} = 180^\circ - 35^\circ 15' 51,8'' = \boxed{144^\circ 44' 8,2''}$$

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valor coincidente con el calculado anteriormente

Angulo rectilíneo " $\varphi_{4-8}$ " del diedro formado por una cara cuadrada y otra octogonal

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{4-8}} = \alpha_4 + \alpha_8 = 77^\circ 14' 8.2'' + 57^\circ 45' 51.8'' = \boxed{135^\circ}$$

También puede obtenerse directamente, así:

$$\boxed{\tan \varphi_{4-8}} = \tan(\alpha_4 + \alpha_8) = \frac{\tan \alpha_4 + \tan \alpha_8}{1 - \tan \alpha_4 \times \tan \alpha_8} = \frac{(3 + \sqrt{2}) + (3 - \sqrt{2})}{1 - (3 + \sqrt{2})(3 - \sqrt{2})} =$$

$$= \frac{6}{1 - 7} = -\frac{6}{6} = \boxed{-1} \quad \text{y haciendo } \alpha_0 = \pi - \varphi_{4-8}, \text{ sera:}$$

$$\tan \alpha_0 = -\tan \varphi_{4-8} = -(-1) = 1 \quad \alpha_0 = 45^\circ$$

$$\boxed{\varphi_{4-8}} = 180^\circ - 45^\circ = \boxed{135^\circ}$$

valor coincidente con el calculado anteriormente,

Angulo rectilíneo " $\varphi_{6-8}$ " del diedro formado por una cara exagonal y otra octogonal

Aplicando la fórmula general [4] (ver lám. 33)

$$\boxed{\varphi_{6-8}} = \alpha_6 + \alpha_8 = 67^\circ 30' + 57^\circ 45' 51.8'' =$$

$$= \boxed{125^\circ 15' 51.8''}$$





También puede obtenerse directamente, así:

$$\begin{aligned} \boxed{\frac{1}{2} \varphi_{6-8}} &= \frac{1}{2} (\alpha_6 + \alpha_8) = \frac{\frac{1}{2} \alpha_6 + \frac{1}{2} \alpha_8}{1 - \frac{1}{2} \alpha_6 \times \frac{1}{2} \alpha_8} = \frac{(1+\sqrt{2})(3-\sqrt{2})}{1 - (1+\sqrt{2})(3-\sqrt{2})} = \\ &= \frac{4}{1 - (3 + 3\sqrt{2} - \sqrt{2} - 2)} = \frac{4}{1 - (1 + 2\sqrt{2})} = -\frac{4}{2\sqrt{2}} = \boxed{-\sqrt{2}} \end{aligned}$$

y haciendo  $\alpha_0 = \pi - \varphi_{6-8}$ , será:  $\frac{1}{2} \alpha_0 = -\frac{1}{2} \varphi_{6-8} = -(-\sqrt{2}) = \sqrt{2} =$

$= 1, 41 42 13 56 \dots$

$$\frac{1}{2} \tan \alpha = 0, 15 05 15 0$$

$\alpha_0 = 54^\circ 44' 8,2''$ , por lo que será:

$$\boxed{\varphi_{6-8}} = 180^\circ - 54^\circ 44' 8,2'' = \boxed{125^\circ 15' 51,8''}$$

Área lateral "S" del arquimediano.

Se compone de la suma de 12 caras cuadradas, 8 hexagonales y 6 octogonales, todas de lado "l".

La apotema de la cara hexagonal, será: (ver lám. 42, b 9)

$$\text{apotema } P_6 = \frac{\sqrt{3}}{2} l$$

La apotema de la cara octogonal, será: (ver lám. 40, b 9)

$$\text{apotema } P_8 = \frac{\sqrt{3+2\sqrt{2}}}{2} l$$

y el área lateral S

$$\boxed{S} = 12 l^2 + 8 \times \frac{6}{2} \times \frac{\sqrt{3}}{2} l^2 + 6 \times \frac{8}{2} \times \frac{\sqrt{3+2\sqrt{2}}}{2} l^2 =$$



$$= (12 + 12\sqrt{3} + 12\sqrt{3+2\sqrt{2}}) l^2 = 12 \left( 1 + \sqrt{3} + \sqrt{\frac{4}{2}} + \sqrt{\frac{2}{2}} \right) l^2 =$$

$$= 12 (1 + \sqrt{3} + \sqrt{2} + 1) l^2 = \boxed{12 (2 + \sqrt{3} + \sqrt{2}) l^2} = 61, 75 \ 51 \ 72 \ 44 \dots l^2$$

### Volumen "V" del arquimedeano

Se compone de la suma de 12 pirámides regulares de base cuadrada y altura " $C_4$ "; de 8 pirámides exagonales de altura " $C_6$ " y de 6 octogonales, de altura  $C_8$ . Su volumen será pues:

$$\boxed{V} = 12 l^2 \times \left( \frac{3+\sqrt{2}}{2} : 3 \right) l + 12 \sqrt{3} l^2 \left( \frac{\sqrt{6}+\sqrt{3}}{2} : 3 \right) l + 12 (\sqrt{2}+1) l^2 \left( \frac{2\sqrt{2}+1}{6} \right) l =$$

$$= \left[ 2(3+\sqrt{2}) + 2\sqrt{3}(\sqrt{6}+\sqrt{3}) + 2(\sqrt{2}+1)(2\sqrt{2}+1) \right] l^3 =$$

$$= (6 + 2\sqrt{2} + 2 \times 3\sqrt{2} + 6 + 2(4 + 2\sqrt{2} + \sqrt{2} + 1)) l^3 =$$

$$= (6 + 2\sqrt{2} + 6\sqrt{2} + 6 + 10 + 6\sqrt{2}) l^3 = (22 + 14\sqrt{2}) l^3 =$$

$$= \boxed{2(11 + 7\sqrt{2}) l^3} = 41, 79 \ 89 \ 89 \ 85 \dots l^3$$

### FIGURA CORPÓREA

Se obtiene por acoplamiento de 12 cuadrados de lado  $l = 23,7 \text{ mm}$ ; de 8 exágonos y 6 octógonos, también

The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

In the second part of the paper the author discusses the question of the structure of the atom in connection with the results of the experiments of Rutherford and his co-workers.

The third part of the paper is devoted to a discussion of the question of the structure of the atom in connection with the results of the experiments of Bohr and his co-workers.

In the fourth part of the paper the author discusses the question of the structure of the atom in connection with the results of the experiments of Heisenberg and his co-workers.

The fifth part of the paper is devoted to a discussion of the question of the structure of the atom in connection with the results of the experiments of Schrödinger and his co-workers.

In the sixth part of the paper the author discusses the question of the structure of the atom in connection with the results of the experiments of Dirac and his co-workers.

The seventh part of the paper is devoted to a discussion of the question of the structure of the atom in connection with the results of the experiments of Pauli and his co-workers.

In the eighth part of the paper the author discusses the question of the structure of the atom in connection with the results of the experiments of Fermi and his co-workers.

The ninth part of the paper is devoted to a discussion of the question of the structure of the atom in connection with the results of the experiments of Einstein and his co-workers.

In the tenth part of the paper the author discusses the question of the structure of the atom in connection with the results of the experiments of de Broglie and his co-workers.



regulares y de igual lado. El acoplamiento deberá hacerse de forma que en cada vértice concurren un cuadrado, un hexágono y un octógono.

En el cuadro sinóptico que damos a continuación, resumimos los resultados analíticos obtenidos anteriormente.

CUADRO SINÓPTICO

Magnitud	Valor exacto	Valor decimal aproximado
$a$	$\frac{\sqrt{13 + 6\sqrt{2}}}{2} \ell$	2. 31 76 11.... $\ell$
$b$	$\sqrt{\frac{6 + 3\sqrt{2}}{2}} \ell$	2. 26 30 33.... $\ell$
$c_4$	$\frac{3 + \sqrt{2}}{2} \ell$	2. 20 71 07.... $\ell$
$c_6$	$\frac{\sqrt{6} + \sqrt{3}}{2} \ell$	2. 09 07 70.... $\ell$
$c_8$	$\frac{2\sqrt{2} + 1}{2} \ell$	1. 91 42 14.... $\ell$
$d_4$	$\frac{\sqrt{2}}{2} \ell$	0. 70 71 07.... $\ell$
$d_6$	1 $\ell$	1. 00 00 00... $\ell$
$d_8$	$\sqrt{\frac{2 + \sqrt{2}}{2}} \ell$	1. 30 65 63.... $\ell$
$m$	$\sqrt{\frac{6(14 + \sqrt{2})}{97}} \ell$	0. 97 64 51... $\ell$
$\alpha_4$	$\operatorname{tg} \alpha_4 = 3 + \sqrt{2}$	$\operatorname{tg} \alpha_4 = 4. 41 42 14...$ $\alpha_4 = 77^\circ 14' 8.2''$
$\alpha_6$	$\operatorname{tg} \alpha_6 = 1 + \sqrt{2}$	$\operatorname{tg} \alpha_6 = 2. 41 42 14...$ $\alpha_6 = 67^\circ 30' 0.0''$
$\alpha_8$	$\operatorname{tg} \alpha_8 = 3 - \sqrt{2}$	$\operatorname{tg} \alpha_8 = 1. 58 57 86..$ $\alpha_8 = 57^\circ 45' 51.8''$
$\varphi_{4-6}$	$\operatorname{tg} \varphi_{4-6} = -\frac{\sqrt{2}}{2}$	$\operatorname{tg} \varphi_{4-6} = -0. 70 71 07$ $\varphi_{4-6} = 144^\circ 44' 8.2''$
$\varphi_{4-8}$	$\operatorname{tg} \varphi_{4-8} = -1$	$\varphi_{4-8} = 135^\circ 0' 0.0''$
$\varphi_{6-8}$	$\operatorname{tg} \varphi_{6-8} = -\sqrt{2}$	$\operatorname{tg} \varphi_{6-8} = -1. 41 42 14...$ $\varphi_{6-8} = 125^\circ 45' 51.8''$
$S$	$12(2 + \sqrt{2} + \sqrt{3}) \ell^2$	61. 75 51 72... $\ell^2$
$V$	$2(11 + 7\sqrt{2}) \ell^3$	41. 79 89 90.... $\ell^3$

The first of these is the fact that the  
 number of cases of disease is not  
 proportional to the number of persons  
 exposed to the disease. This is  
 because the disease is not equally  
 contagious to all persons.

Table showing the results of the experiments on the contagiousness of the disease		
Number of persons exposed to the disease	Number of persons who became diseased	Percentage of persons who became diseased
10	2	20
20	4	20
30	6	20
40	8	20
50	10	20
60	12	20
70	14	20
80	16	20
90	18	20
100	20	20

PROCESO GRÁFICO - ANALÍTICO

Después del cálculo de las magnitudes principales, vamos a proceder, en la lámina 43, a la representación gráfica del Arquimédiano XI.

Para su trazado nos valdremos de cotas calculadas por las fórmulas anteriores, y de procesos gráficos.

Con este objeto, calculemos previamente las siguientes magnitudes:

$$l_{x1} = \text{Dato del ejercicio} = 23,7 \text{ mm}$$

$$a = 2,317611... \times 23,73 = 55,0 \text{ mm}$$

$$b = 2,263033... \times 23,73 = 53,7 \text{ mm}$$

$$c_4 = 2,207107... \times 23,73 = 52,4 \text{ mm}$$

$$c_6 = 2,090770... \times 23,73 = 49,6 \text{ mm}$$

$$c_8 = 1,914214... \times 23,73 = 45,4 \text{ mm}$$

$$d_4 = 0,707107... \times 23,73 = 16,8 \text{ mm}$$

$$d_6 = 1,000000... \times 23,73 = 23,7 \text{ mm}$$

$$d_8 = 1,306563... \times 23,73 = 31,0 \text{ mm}$$

El orde de operaciones del trazado gráfico (lámin. 43) es el siguiente:

1° Situar el centro O, de coordenadas O(72, 72, 85) mm.

2° Dibujar, en I, II y III las proyecciones de la esfera circunscrita, de radio  $a = 55 \text{ mm}$ .

The following information is for your information only. It is not intended to be used as a basis for any action. The information is for your information only. It is not intended to be used as a basis for any action. The information is for your information only. It is not intended to be used as a basis for any action.

1. 100	2. 100	3. 100	4. 100
5. 100	6. 100	7. 100	8. 100
9. 100	10. 100	11. 100	12. 100
13. 100	14. 100	15. 100	16. 100
17. 100	18. 100	19. 100	20. 100
21. 100	22. 100	23. 100	24. 100
25. 100	26. 100	27. 100	28. 100
29. 100	30. 100	31. 100	32. 100
33. 100	34. 100	35. 100	36. 100
37. 100	38. 100	39. 100	40. 100

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3° Representar en I, II y III, las caras octogonales superior 1 al 8, e inferior 41 al 48, supuesto el poliedro colocado con dichas caras paralelas a II, y un lado perpendicular a I (utilizase la cota " $c_4$ " en I y III, y la " $d_4$ " en II).

4° Representar en I, II y III las caras octogonales anterior (15-24-25-34-35-26-17); posterior (12-21-30-39-38-29-20-11); lateral izquierda (13-22-31-40-33-32-23-14) y lateral derecha (10-19-28-37-36-27-18-9), siguiendo el mismo proceso que el dado en el anterior trazado 3°.

5° Completar las proyecciones en I, II y III de las aristas restantes (paralelas a los ejes principales o a sus bisectrices).

Obrórese que por la posición elegida en la representación de este arquimedianos, son iguales las proyecciones de sus tres vistas, aun cuando lógicamente es distinta la numeración de sus vértices.



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the sum of \$100.00 for the year 1911  
and for the year 1912

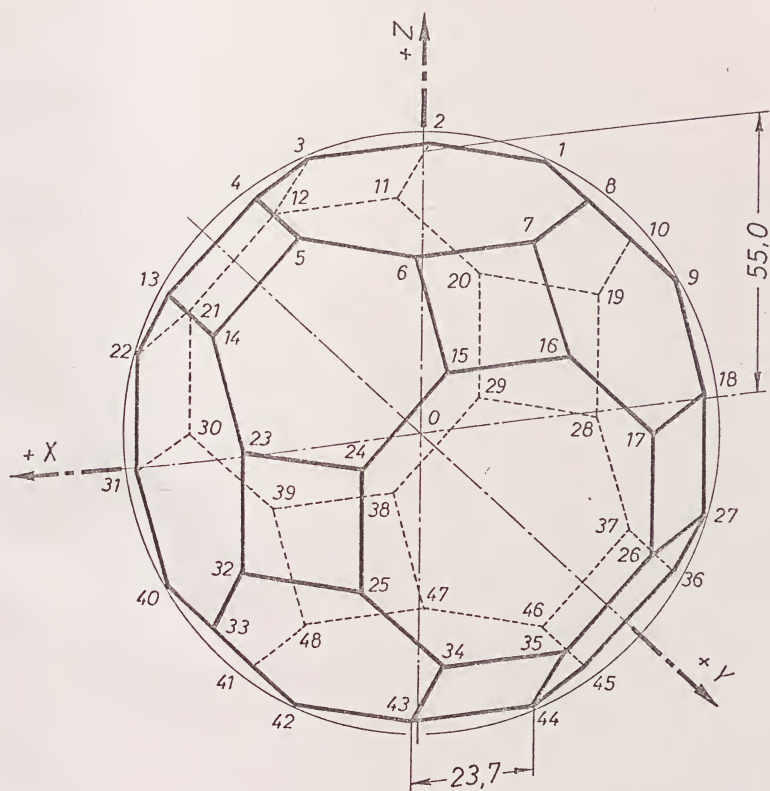
and for the year 1913  
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and for the year 1945  
and for the year 1946  
and for the year 1947  
and for the year 1948  
and for the year 1949  
and for the year 1950  
and for the year 1951  
and for the year 1952



Arquimiliano XI







